MONTHLY NOTICES

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ROYAL ASTRONOMICAL SOCIETY

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MEETING OF 1941 OCTOBER 10

Professor S. Chapman, President, in the Chair

Thirty-two presents were announced as having been received since the last ordinary meeting, including:

Smart (W. M.), Sea and Air Navigation (Halley Lecture, 1941).

The President announced that the next Ordinary Meeting would be held on 1942 March 13, and that the Annual General Meeting would be held in April.

He further announced that the Library of the Society would be open to Fellows on the second Friday of each month during the winter.

THE SOLAR PARALLAX AND THE MASS OF THE MOON FROM OBSERVATIONS OF EROS AT THE OPPOSITION OF 1931

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(Received 1941 October 10)

1. At the opposition of 1931, the asteroid 433 Eros approached the Earth to a distance of 16·2 million miles, being its closest approach since its discovery in 1898. This provided a particularly favourable occasion for the determination of the solar parallax, one of the fundamental constants in astronomy. The path of Eros in the sky near opposition formed a large arc, stretching from declination $+44°$ to $-26°$, enabling observations to be obtained at observatories in both hemispheres. Independent determinations of the solar parallax thus became possible from the observations in right ascension and from those in declination; in right ascension, the solar parallax could be determined from the comparison between observations taken at easterly and westerly hour-angles with the same instrument; in declination, it could be determined from comparisons between observations made practically simultaneously at two observatories, one in the northern and the other in the southern hemisphere. The same observations, provided they were extended over a sufficient period of time, would provide a determination of the ratio of the mass of the Moon to the mass of the Earth; apart from the intrinsic importance of an accurate determination of the mass of the Moon, a more accurate determination of this mass was of importance because of the known, but unexplained, discordance between the observed value of the constant of nutation and the value inferred from the observed values of the constant of precession and the mass of the Moon.

2. At the General Assembly of the International Astronomical Union in 1928, a Solar Parallax Commission was appointed, with the writer as President, to make arrangements for the co-operative effort to secure the observations and for their subsequent reduction and discussion. The results of the discussion of the observations that were obtained are being published in some detail in the *Memoirs* of the Society. The present communication summarises the results given in this *Memoir*.

3. The first step was the determination of the positions, on a homogeneous system, of a system of primary reference stars distributed along the apparent path of Eros, sufficient in number to serve as a basis for the determination of the position of Eros from photographs of astrographic scale, covering $2° \times 2°$, centred at any point of the path of Eros, and bright enough for observation with meridian instruments. These stars were selected by Professor Kopff and were widely observed at the following observatories: Berlin-Babelsberg, Besançon, Cincinnati, Cape, Greenwich, Hamburg, Heidelberg, La Plata, Leiden, Lick, Padova, Uccle and Washington. The observations were discussed with great care at the Rechen-Institut, Berlin, under the direction of Professor Kopff, and the observed positions were reduced to a system as nearly homogeneous as possible and combined to form a general catalogue of primary reference stars.

4. The primary reference stars, which averaged ten to twelve on an astrographic plate, were insufficient in number for the reduction of plates obtained with instruments of longer focal length. For the determination of the positions of a sufficient number of fainter or secondary stars to serve as reference stars for the reduction of plates taken with the longest focus instruments, four special series of overlapping plates, covering the path

of Eros, were obtained at the Bergedorf, Leipzig, Greenwich and Cape Observatories. As the Cape series was the only one to extend south of $-14°$, the positions of the southern secondary reference stars were strengthened by the incorporation of the positions derived from some plates, covering $5° \times 5°$, obtained with a wide-angle lens camera at the Lick Observatory. The definitive catalogue of secondary reference stars, containing the positions of 5823 stars on the basis of the system of primary star places, was prepared by Mr. Melotte and the writer.

5. The computation of the definitive ephemeris of Eros for the interval 1930 October 0 to 1931 June 3 was undertaken by Dr. G. Witt, the discoverer of Eros. The first stage in this work was the computation by special perturbations of an accurate ephemeris of the Sun, the positions of the Sun computed from Newcomb's Tables not being sufficiently accurate for the purpose. From this ephemeris, rectangular co-ordinates of the Sun were obtained, which are believed to be free from computational errors exceeding two units in the ninth decimal place. The second stage was the derivation of improved orbital elements of Eros; these were based on 34 normal places, derived from observations at the oppositions of Eros from 1898 to 1931, the four normal places for the opposition of 1931 being obtained from preliminary results of the positions of Eros that were made available to Dr. Witt. Each normal place gave an equation of condition for differential corrections to the orbital element, the 34 equations being solved by least squares. Revised elements for the epoch and osculation 1898 August 2·0 M.T. Berlin, and for the equator and equinox of 1900, were thus derived. These were brought forward with the application of planetary perturbations to the new osculation epoch 1931 January 18·0 U.T., and referred to the equator and equinox of 1930·0. The rectangular co-ordinates of Eros were formed at two-day intervals by numerical integration, reduced to the barycentre, smoothed, interpolated to daily intervals, converted to R.A. and Dec., smoothed again and then interpolated to derive the barycentric values of the R.A. and Dec. of Eros at intervals of 0·25 day. The reductions from the centre of the Earth to the barycentre are separately tabulated in the ephemeris.

6. The observational material used includes results from the following observatories: Pulkovo, Bergedorf, Radcliffe (Oxford), Greenwich, Leipzig, Uccle, Prague, Dearborn, Van Vleck, Allegheny, Washington, Leander McCormick, Catania, Lick, Algiers, San Fernando, Tokyo, Zô-Sè, Johannesburg (Union Observatory), Johannesburg (Yale), Cordoba, Cape of Good Hope, La Plata, and Melbourne. At some of these observatories, more than one telescope was used, so that results of observations with 30 different telescopes are available. The results from a few other observatories have not been used, either because they were not available in time, or for other reasons. This material is all photographic; micrometric measures were made at various observatories, but as the results do not compare in accuracy with the photographic measures and are more affected by errors in the places of the reference stars, they have not been used. The discussion is based on the results from 1392 plates obtained at 18 northern observatories and 1455 plates obtained at 6 southern observatories, a grand total of 2847 plates.

7. The observed places of Eros were corrected for parallax, using $8''·806$ as the basic value of the solar parallax, and reduced to barycentric positions by the application of the barycentric corrections tabulated in the ephemeris. The tabular places were interpolated from the ephemeris and the residuals between the observed and tabular positions provided the basic material for the discussion. The residuals from all the observations were plotted, both for R.A. and Dec., on large scale diagrams and curves were drawn as smoothly as possible to represent their general run. The daily rates of change of the tabular errors in R.A. and Dec. for each day were read from these curves.

8. Small systematic differences were found to exist between the results from the various series. For the combination of the various results it was necessary to determine these systematic differences and to derive an appropriate system of weighting. The systematic differences were derived from intercomparisons of the various series, by differencing the

mean residuals of observations on the same or adjacent nights, corrections being applied for the change in tabular residuals during the small difference of time in the observations of the two series being compared. The longer series were first intercompared in pairs and systematic corrections, referred to an arbitrary zero, were derived for these series. The shorter series were then compared with several of the longer series, the previously determined corrections to the latter series being applied. The corrections in R.A. were finally adjusted so that the general mean for all series was zero. In dealing with the declination results, it was necessary to discuss the northern and southern observations separately, because the determination of the solar parallax from the declination observations depends upon the comparison between northern and southern observations. The corrections in declination were therefore adjusted so that the means for the northern and southern series were separately zero. The systematic corrections finally applied range from $+0^s.015$ to $-0^s.006$ in R.A. and from $+0''.14$ to $-0''.09$ in Dec.

A preliminary examination of the material showed that the weight of several plates with the same instrument on the same night was not proportional to the number of plates. It was therefore assumed that for each instrument the error of a single plate consisted of two parts: an error common to the night, $\pm \epsilon_0$, and the accidental error of a single plate, $\pm \epsilon_1$. The squared error of the mean of n plates taken on the same night is therefore $\epsilon_0^2 + \epsilon_1^2/n$. The appropriate values of ϵ_0 and ϵ_1 for each of the longer series of observations were derived by least squares. Intercomparisons of the discordances for each series separately between the residuals obtained on the same night give values of ϵ_1^2. Intercomparisons between the mean residuals for adjacent nights with the same instrument (corrections being applied for change of tabular error with time), and between the mean residuals for the same nights with two different instruments (corrections being applied for the systematic difference between the two instruments), give relations involving ϵ_0^2 and ϵ_1^2 for one or both instruments. A large number of intercomparisons between the longer series were made, the resulting equations being combined in a single least squares solution. The weights for the longer series having been thus obtained, those for the shorter series were derived by comparing them with several of the longer series. It was found that, in general, the values of ϵ_0, the errors common to the night, were between one-half and one-quarter of the corresponding values of ϵ_1. The range in weights for a single plate in both R.A. and Dec. was very considerable, the extreme values of ϵ_1 being $\pm 0^s.0026$ and $\pm 0^s.0500$ in R.A. and $\pm 0''.039$ and $\pm 0''.536$ in Dec.

9. Solutions for the solar parallax were made first for each instrument for which observations on the same night with a sufficient range in parallactic displacement in R.A. were available for at least several nights. In general, comparisons were restricted to evening and morning observations during the same night, but where suitable comparisons between unbalanced observations on adjacent nights were available these were also included. In all cases allowance was made for change of tabular error with time in the interval covered by the comparisons. These determinations of the solar parallax are independent of systematic corrections required by individual instruments, but any systematic difference of instrumental origin between observations east and west of the meridian (such as might be caused, for instance, by an imperfect adjustment for plate tilt) enters directly into them.

The corrections to the basic value, $8''.806$, of the solar parallax derived from these individual series; the probable errors of these corrections, inferred from the internal agreement between the separate comparisons; the number of comparisons on which each determination is based and the relative weights of each correction, are summarised in the table.

Summary of Separate Determinations of Correction to Solar Parallax 8″·806

Instrument	$\Delta\pi_\odot$	No. of Comparisons	Relative Weight
	″ ″		
Cape 13-in.	−0·0174 ± 0·0016$_5$	57	36·7
Cape 24-in.	− ·0158 ± ·0017$_5$	46	32·6
Cordoba 13-in.	− ·0132 ± ·0024	27	16·4
Allegheny	− ·0040 ± ·0035	17	8·1
Van Vleck	− ·0384 ± ·0048	16	4·3
Dearborn	+ ·0010 ± ·0061	30	2·7
Zô-Sè	− ·0192 ± ·0049	8	4·2
Union Observatory	− ·0142 ± ·0074	16	1·8
Lick	− ·0173 ± ·0080	12	1·6
Melbourne	− ·0111 ± ·0082	11	1·5
Greenwich 13-in.	− ·0412 ± ·0078	22	1·6
,, 26-in.	− ·0240 ± ·0115	11	0·8
McCormick	− ·0199 ± ·0091	12	1·2
Washington	+ ·0100 ± ·0101	9	1·0
Tokyo	− ·0601 ± ·0183	21	0·3
Catania	− ·0195 ± ·0336	40	0·1

The weighted mean value of the correction to the solar parallax is −0″·0160. The probable error for unit weight deduced from the discordances of the individual determinations is ± 0″·0143, whereas the weights were assigned on the basis of a probable error for unit weight of ± 0″·0100. This difference probably indicates the existence of systematic errors of instrumental origin in some of the series.

Adopting the larger probable error, the value for the solar parallax derived from these determinations is 8″·7900 ± 0″·0013.

10. The internal consistency of the results can be examined by subdividing the material in different ways: (*a*) The results with the Cape Astrographic, Cape 24-inch and Cordoba Astrographic have a preponderating weight. Systematic errors in these series would have an appreciable effect on the mean result. Separating these three instruments from the others we obtain:

Cape 13-in., Cape 24-in. and Cordoba	$\pi_\odot = 8''{\cdot}7900 \pm 0''{\cdot}0015$
13 other instruments.	$\pi_\odot = 8''{\cdot}7901 \pm 0''{\cdot}0026$

(*b*) Some determinations are based entirely on primary star positions, others entirely or mainly upon secondary star positions. Dividing into two groups, according to whether primary or secondary reference stars were used, the mean results are:

Primary star places (6 insts.)	$\pi_\odot = 8''{\cdot}7894 \pm 0''{\cdot}0019$
Secondary ,, ,, (10 insts.)	$\pi_\odot = 8''{\cdot}7907 \pm 0''{\cdot}0019$.

(*c*) Five of the determinations were made with visual instruments or their equivalent, *i.e.* using yellow sensitive plates in conjunction with yellow filters. The remaining determinations were made with photographic instruments and ordinary plates. The separate mean results are:

Photographic (11 insts.)	$\pi_\odot = 8''{\cdot}7903 \pm 0''{\cdot}0014$
Visual (5 insts.)	$\pi_\odot = 8''{\cdot}7876 \pm 0''{\cdot}0043$

The material has therefore a satisfactory degree of stability.

11. A considerable wastage of material is inherent in the above method of solution. Not only are unbalanced observations with these instruments excluded, but also observations with other instruments have not been utilised at all—either because there were too few nights with suitably balanced observations or because there was not a sufficient

range in parallax factor. New solutions were therefore made, night by night, using all available material. Corrections were applied for change in tabular error with time, to reduce the observations on each night to the same epoch, and the systematic corrections to the various instruments were applied. In some cases, where the weights were small, two or more nights were combined. 129 separate determinations of the correction to the solar parallax were thus obtained. The mean values of the solar parallax and its probable error, inferred from the discordances between the separate values, are $8''\cdot7875 \pm 0''\cdot0009$.

In order to check the internal consistency of the material, it was divided into four groups of approximately equal weight. The values of the solar parallax from these four groups are $8''\cdot7874$, $8''\cdot7871$, $8''\cdot7899$, $8''\cdot7855$, which are in close accordance.

12. The range in parallactic displacement in declination in the course of the night is insufficient to enable a reliable determination of the solar parallax to be derived from observations in declination with a single instrument. But the parallactic displacements in declination have opposite signs at two places, for one of which Eros is north and for the other south of the zenith, enabling the parallax to be determined with high weight from the comparison between observations at northern and southern observatories. Observations on or near the meridian, when the parallactic displacement in R.A. is zero or small, can be utilised. In such comparisons, it is necessary to apply the systematic corrections to the various instruments.

A number of determinations of the solar parallax were made by comparing results in declination in this way. The results of these determinations are given below. They are not all independent, as some observations are used in several intercomparisons. The individual solutions are of interest, however, as they show the general degree of accordance obtained.

Determinations of Solar Parallax from Comparisons between Northern and Southern Observations in Declination

Instruments	Solar Parallax	No. of Nights
	″ ″	
Radcliffe; Cape 24-in.	8·7934 ± 0·0026	9
Greenwich 26-in.; Cape 24-in.	8·7935 ± ·0040	9
Radcliffe and Greenwich 26-in.; Cape 24-in. and Yale	8·7908 ± ·0026	13
Greenwich 13-in.; Cape 13-in.	8·8023 ± ·0067	8
Dearborn; Cape 24-in.	8·7906 ± ·0033	28
Van Vleck; Cape 24-in. and Yale	8·7874 ± ·0039	17
Algiers; Cape 13-in.	8·7886 ± ·0028	49
Algiers; Cordoba	8·7865 ± ·0033	30
Allegheny; Cape 24-in. and Yale	8·7840 ± ·0037	22
Washington; Cape 24-in. and Yale	8·7870 ± ·0058	14
McCormick; Cape 24-in. and Yale	8·7865 ± ·0038	17
Lick; Cape 13-in.	8·7862 ± ·0036	12
Lick; Cape 24-in.	8·7867 ± ·0043	12
Lick; Cordoba	8·7909 ± ·0086	9

The straight mean of the above values is $8''\cdot7890$; the weighted mean is $8''\cdot7883$. These results are in close accordance with the results from the R.A. observations.

A general solution from the observations in declination was made by combining all the material and determining the solar parallax from the observations grouped into 24-hour intervals, or into groups of two or three days in cases where the weight was otherwise insufficient. 71 separate determinations were thus made, giving a mean result of $8''\cdot7907 \pm 0''\cdot0011$. This result includes, of course, the whole of the material used in the individual comparisons above. The probable error is derived from the internal agreement of the results.

The systematic corrections applied in this solution reduce each northern instrument

to the general mean of all the northern instruments and each southern instrument to the general mean of all the southern instruments. The correction terms were kept separate during the solutions, in order that their effect on the final value of the solar parallax could be determined; without the application of the corrections, the mean value of the solar parallax from the declination observations becomes 8″·7886.

13. The final results for the solar parallax may therefore be summarised as follows:—

R.A. Observations.

(*a*) Mean value from 16 separate determinations,	8″·7900 ± 0″·0013
(*b*) Value from all observations combined,	8″·7875 ± 0″·0009

Dec. Observations.

(*c*) Value from all observations combined,	8″·7907 ± 0″·0011

Of these results (*c*) is entirely independent of (*a*) or (*b*). The latter are not independent, as all the material used in (*a*) is included in (*b*), but (*a*) does not involve the application of any systematic corrections. The three determinations are adequately represented by a value of the solar parallax of

$$8''\cdot790 \pm 0''\cdot001$$

which is adopted as the final value resulting from the observations of Eros at the opposition of 1931.

14. The probable error assigned to the final value of the solar parallax is based on the internal accordance of the results. The question arises whether it is a true measure of the uncertainty of the determination or whether there is a systematic source of error affecting all the observations. A possible source of such error is differential atmospheric dispersion. The parallactic displacement of Eros is in a direction away from the zenith; refraction displaces the stars and Eros towards the zenith and by a greater amount the bluer the colour of the object. If, then, Eros is redder than the average of the comparison stars, it will have a differential displacement away from the zenith relative to the system of comparison stars. This will be interpreted in the reductions as a parallactic displacement, so that the determined displacements will be too large. In consequence, too large a value of the solar parallax will be inferred. The mean spectral type of the comparison stars is about G5, whilst measurements of the colour of Eros made at Greenwich, Uccle and the Cape gave a colour corresponding to spectral type G7 or G8. Eros is thus slightly redder than the mean of the comparison stars.

The effects of colour differences in the actual observations are, however, of a somewhat complex nature and depend upon the design of the objective (and, in particular, upon its colour curve), upon the properties of the particular photographic emulsion employed, upon the length of exposure, and upon various other factors in addition to the spectral type of the star. It is known, for instance, that in measurements of effective wave-lengths of stars with a coarse wire diffraction grating (when the dispersion normally used corresponds approximately to atmospheric dispersion at an altitude of a few degrees), the measured dependence of effective wave-lengths on spectral type is much greater, for the same instrument, when panchromatic plates are used than when ordinary plates are used. For the same photographic emulsion, the range obtained with a reflector is about double that obtained with a refractor.

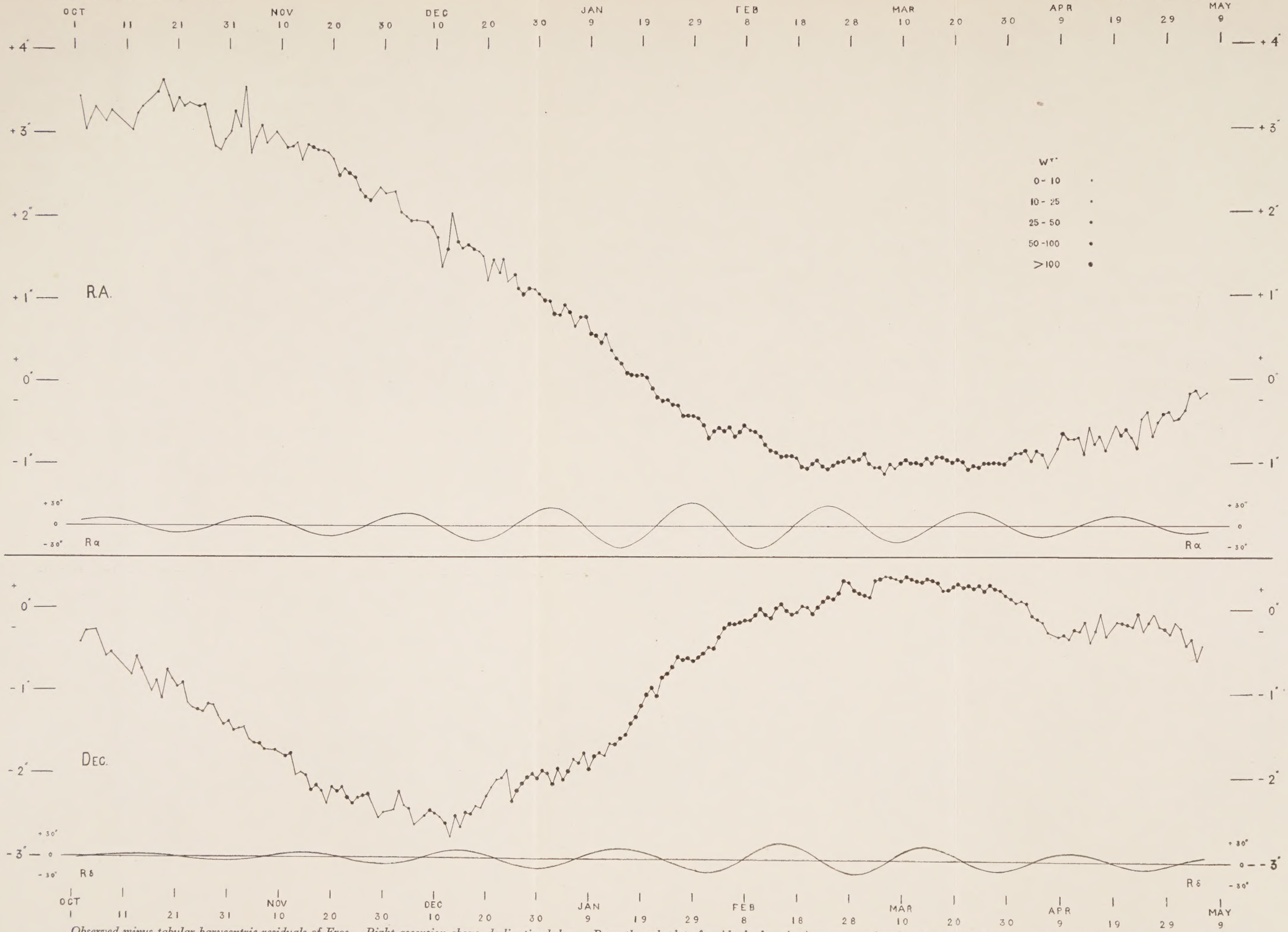
The measured effective wave-lengths depend upon the length of exposure and the magnitude of the star, or, in other words, upon the image density. With short exposures, only the wave-lengths near the wave-length of maximum intensity in the energy distribution curve of the star are effective in producing an image on the plate. With increase in the length of exposure, other wave-lengths become effective, but, because of the asymmetry of the energy distribution curve, the growth of the image is asymmetrical. The resulting dependence upon image density is greater for blue than for red stars.

Curvature of field is also of importance. If a refractor of the normal astrographic type is focused to give the sharpest images at the centre then, at the edge of the plate, the rays that are brought to a sharp focus are not those that are photographically the most effective, but are of a somewhat longer wave-length. The measured effective wave-length depends upon the position of the star on the plate, the stars at the edge appearing redder than those at the centre of the same spectral type, the effect being greater for blue than for red stars. A detailed investigation with the Cape astrographic telescope showed that the distance effect was more than sufficient to counterbalance the effect of Eros being slightly redder than the stars. With the Lick wide-angle lens camera, on the other hand, no distance effect could be detected.

The conclusion is that the effects due to differences of colour are complicated in character and when the results from a large number of instruments are combined they are no longer purely systematic, but tend to assume in part a random character. In so far as any residual systematic effect remains, it should be revealed by a correlation between the determined values of the solar parallax night by night and the distance of Eros; for the atmospheric dispersion effects are independent of the distance of Eros, whereas the parallactic displacements vary inversely with the distance. An investigation of the correlation of the solar parallax with distance was made for both R.A. and Dec. observations. The deduced effects, which were comparable with their probable errors, were of opposite sign in the two co-ordinates. The conclusion is that the general effects arising from colour differences between Eros and the comparison stars are small. This conclusion is supported by the agreement, within the limits of their probable errors, between the mean values of the solar parallax derived from photographic and visual instruments; atmospheric dispersion effects at the mean wave-length of the observations with the visual telescopes (using yellow filters in conjunction with yellow sensitive plates) are only two-fifths as great as the effects at the mean wave-length of the observations with the photographic telescopes.

15. For the determination of the constant of the lunar equation, the tabular errors of the ephemeris in R.A. and Dec. were derived for each day by the combination, with their appropriate weights, of the results from each instrument, the parallactic displacements being reduced to the basis of the value, $8''{\cdot}790$, of the solar parallax. These errors are plotted in the diagram in which the relative weights of the individual values are indicated by the size of the plotted points. The weights were assigned on the basis of a probable error for unit weight of $\pm 0''{\cdot}32$. It will be noted that there are very few days in the interval, from October 1 to May 7, covered by the observations, for which no observations are available; the weights at the beginning are low but gradually increase, and from December 25 to the end of March the observations are uniformly of high weight. The general run of the residuals in each co-ordinate can be represented by a smooth curve, but for a satisfactory representation it is necessary to superpose small irregular fluctuations on this smooth curve. Below each curve is plotted, on a smaller scale, the barycentric corrections. The effect of the adoption, in computing these corrections, of an incorrect value of the mass of the Moon will be to leave a residual fluctuation in the tabular errors, which is proportional to the barycentric correction. It will be readily seen, from inspection of the curves, that the short-period fluctuations in the tabular residuals about their general run are not closely correlated with the lunar equation. They undoubtedly arise in the main from inhomogeneities in the system of reference star places. It will be noticed that the fluctuations are more marked in the residuals in declination than in those in right ascension. This is to be expected, because the motion of Eros, near opposition, was much more rapid in Dec. than in R.A. The smoothness of the residuals near the stationary point in March also indicates that the fluctuations are due to errors in the reference star places.

16. These inhomogeneities in the system of star-places complicate the determination of the constant of the lunar equation and entail a considerable loss of weight. Though



Observed minus tabular barycentric residuals of Eros. Right ascension above, declination below. Beneath each plot of residuals there is given, on a reduced scale, the tabular barycentric corrections (lunar equation).

H. Spencer Jones, The Solar Parallax and the Mass of the Moon from Observations of Eros at the Opposition of 1931.

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the general run of the residuals in either co-ordinate can be represented by an expansion in powers of the time, the deviations of the individual points from any such representation are much greater than the uncertainties of their determination. The only possible procedure was to deal with short stretches of the residuals in succession, representing the residuals over a single period of the lunar equation by a smooth curve, on which a lunar equation term is superposed. If a fluctuation arising from inhomogeneities in the reference star places should happen to coincide approximately in phase with the lunar equation term, a spuriously large correction to the constant of the lunar equation will be obtained; but in the combination of the results from a dozen or so of such stretches, these effects will tend to be of a random nature.

The residuals within a couple of days of a zero of the lunar equation were combined to give weighted mean values of the residuals at the times of each zero of the lunar equation. These were adopted as fiducial points in the solutions, because they are independent of the lunar equation term. Several different solutions were made. In each solution, complete periods of the lunar equation were dealt with in succession; thus except for the first and last half-period, each half-period entered into two solutions and was accordingly given half weight. Particulars of the solutions are as follows:—

Solution A.—The residuals (for a complete period of the lunar equation) were represented by the expression

$$a + bT + cT^2 + d\frac{R}{10},$$

where T denotes the time, measured from a mid-epoch, and R is the tabular barycentric reduction. The constants a, b and c were determined from the three consecutive zeros of the lunar equation, at which R is zero. Using the values so derived, d was obtained by a least squares solution, with each residual being given its assigned weight.

Solution B.—The previous solution does not look outside a complete period of the lunar equation and, when extrapolated outside the extreme zeros used, there is apt to be a rapidly increasing departure of the computed from the observed residuals. In solution B, the observations over an interval of five consecutive zeros of the lunar equation were represented by the expression

$$a + bT + cT^2 + c'T^3 + d\frac{R}{10}.$$

The constants a, b, c, c' were determined by least squares adjustment from the residuals at the five zeros. Using these values, d was then determined by least squares from the observations between the second and the fourth of the zeros.

Solutions C and D.—Because of the inhomogeneities in the star-place system, the precision with which the observed positions of Eros are determined in relation to this system is greater than the precision with which they are determined in relation to the true star-places. The inhomogeneities in the adopted star places reduce the effective weights. Investigation showed that the probable error introduced by the inhomogeneities is approximately one-tenth of the adopted probable error for unit weight, *i.e.* about $\pm 0''{\cdot}032$. A new series of reduced weights were accordingly formed, using the expression $100w/(100 + w)$. Solutions C and D were repetitions of Solutions A and B respectively, using the reduced weights.

Solution E.—The representation of the residuals, particularly in declination, by the preceding solutions was somewhat unsatisfactory, even over the short interval covered by a period of the lunar equation. A graphical method of solution was therefore tried. As smooth a curve as possible was drawn to represent the run of the residuals in each co-ordinate, and to pass through the fiducial points corresponding to the zero values of the lunar equation. In drawing this curve, no attention was paid to the minor fluctuations and irregularities in the residuals, but where there was an appreciable and rapid run,

the curve was made to follow its general course. The differences between the observations and the smooth curves were represented by the expression $dR/10$. The values of d for each complete period of the lunar equation were derived, using the system of reduced weights.

Solution F.—The solution was based upon the curves drawn for the preceding solution, but the differences between the observations and the smooth curves were represented by the expression $a+dR/10$, the values of a and d being found by least squares for each period of the lunar equation, with the system of reduced weights.

17. The values of d derived for each period of the lunar equation in each of the six solutions are given in the *Memoir*. It is sufficient here to give the weighted mean values, together with their probable errors, which are in all cases determined from the discordances between the values for the several periods of the lunar equation.

Summary of Solutions for Correcting Factor d

Solution	R.A.	Dec.
A	-0.0133 ± 0.0026	-0.0182 ± 0.0077
B	$-\ .0138 \pm\ .0025$	$-\ .0209 \pm\ .0072$
C	$-\ .0138 \pm\ .0027$	$-\ .0240 \pm\ .0086$
D	$-\ .0138 \pm\ .0029$	$-\ .0275 \pm\ .0083$
E	$-\ .0137 \pm\ .0025$	$-\ .0092 \pm\ .0042$
F	$-0.0129 \pm\ .0023$	$-0.0102 \pm\ .0042$

It will be noted that in R.A. the two graphical solutions give practically the same value of the correcting factor, with the same probable error, as the analytical solutions. In Dec., on the other hand, the value of d and its probable error are both appreciably reduced by the graphical solutions. This change reflects the extent to which the effects of the inhomogeneities in the star-place system have been incorrectly interpreted as a lunar equation effect. The graphical solutions for the declination results are considered to give a more reliable value of the correcting factor, than the analytical solutions.

The finally adopted values for the correcting factor, which, in the case of the declinations, are based to some extent upon judgment of the relative validity of the various solutions, are:—

From right ascensions $d = -0.0136 \pm 0.0026$
From declinations $d = -0.0123 \pm 0.0050$

These two determinations are entirely independent and may be combined in accordance with the assigned probable errors. The final value of the correcting factor is therefore

$$d = -0.0133 \pm 0.0023.$$

18. The reductions to the barycentre, tabulated in the ephemeris, are applied to the observed geocentric positions of Eros to reduce them to the barycentre. We have found that the residuals obtained by subtracting the tabular position from the observed position so corrected contains the term $\frac{1}{10}dR$. In other words, the tabular values of R, and hence also the adopted constant of the lunar equation, require to be multiplied by the factor $(1-\frac{1}{10}d)$.

The constant of the lunar equation, L, is defined by

$$L = \frac{\mu}{1+\mu}\,\frac{\pi_\odot}{3422{\cdot}526 \sin 1''},$$

where $\mu E = M$; and E, M denote the masses of the Earth and Moon respectively.

The basic values used in the ephemeris are

$$\pi_\odot = 8''{\cdot}806, \qquad \mu^{-1} = 81{\cdot}53,$$

corresponding to

$$L = 6''{\cdot}4305.$$

The value derived from the Eros observations is accordingly

$$L = 6''\cdot4305[1 + \cdot00133 \pm \cdot00023]$$
$$= 6''\cdot4390 \pm 0''\cdot0015.$$

Using the value for $\pi_\odot$ derived in the present discussion, $8''\cdot790 \pm 0''\cdot001$, we obtain for the ratio of the mass of the Earth to the mass of the Moon,

$$E/M = 81\cdot271 \pm 0\cdot021,$$

where the assigned probable error includes the effects of the probable errors of the determination both of the solar parallax and of the constant of the lunar inequality.

19. From the value of the mass of the Moon, derived in the preceding section and the observed value of the constant of precession, it is possible to compute the constant of nutation. Adopting $5493''\cdot156$ for the constant of precession, the computed value is $9''\cdot2272 \pm 0''\cdot0008$. This is considerably larger than the adopted value of the constant, $9''\cdot210$. The new determination of the mass of the Moon has not removed the discordance referred to in paragraph 1, but has, in fact, rather increased it.

The possible adjustment of constants to remove this discordance was discussed by Brouwer, in editing and completing the unfinished paper by the late W. de Sitter, "On the system of Astronomical Constants." * Brouwer gave three alternative sets of values of the constants: the third, which he regarded as the most satisfactory, adopted a value of $6''\cdot4068$ for the constant of the lunar equation. To this value of the constant there would correspond a value of $+\cdot0368$ for the correcting factor d, determined in paragraph 17. The value there derived differs from $+\cdot0368$ by more than twenty times its probable error.

A general adjustment of all the relevant data was therefore attempted. The data used were the following: the values of the solar parallax and the constant of the lunar equation, as determined from the 1931 Eros observations; the constant of precession; the constant of nutation, for which the value of $9''\cdot2134 \pm 0''\cdot0042$, obtained from the Cookson telescope latitude variation observations at Greenwich, 1911–1931, was assumed; the quantity $\mathcal{J} = \frac{3}{2}(C-A)/Eb^2$, deduced from the observed motions of the Moon's perigee and node; the coefficient of $\sin^2\phi$ in gravity, as recently derived by Jeffreys from all the observational data; the constant of the lunar parallax, given by the Greenwich-Cape determination.

The least squares adjustment of the resulting equations of condition gave the following values of relevant constants:—

Solar Parallax,	$8''\cdot7904 \pm 0''\cdot0010$
Reciprocal of mass of Moon,	$81\cdot288 \pm 0\cdot021$
Constant of lunar inequality,	$6''\cdot4379 \pm 0''0018$
Constant of nutation,	$9''\cdot2266 \pm 0''0008$
Reciprocal of oblateness of the Earth,	$296\cdot776 \pm 0\cdot054$

The values of the solar parallax and of the mass of the Moon are practically unchanged by the general adjustment. The value of the constant of nutation is again much larger than the observed value. The equation of condition based on the nutation constant contributed more than half to the weighted sum of the squares of the residuals and it is again the observed value of this constant that stands out as discordant.

20. The derivation of the mass of the Moon from the constant of the lunar equation involves the solar parallax. If it is supposed that the solar parallax derived from the Eros observations is subject to some unexplained source of systematic error, it is possible from the constant of the lunar equation, determined from the Eros observations, and from the observed constant of nutation to infer the solar parallax. Assuming $9''\cdot210$ for the

* *B.A.N.*, **8**, No. 307, 1938.

constant of nutation, the solar parallax inferred in this way is 8″·841. This is an impossible value and emphasises the incompatibility between the constant of the lunar equation, as determined from the Eros observations, and the generally accepted value of the constant of nutation.

The determination of the constant of nutation is usually considered to be free from the danger of systematic error. This does not seem to be the case: in the two most reliable single determinations, that made by Przybyllok based on the international latitude-variation observations, and that made by the writer from the Greenwich latitude-variation observations, there was a marked dependence of the determined constant upon the right ascension of the stars. An accurate determination of the constant of nutation, from observations planned specially for this purpose, is much to be desired.

Summary

1. The observations of Eros, obtained around the opposition of 1931, have been discussed and determinations of the solar parallax and of the constant of the lunar equation have been made.

2. The solar parallax is 8″·790 ± 0″·001. Independent determinations from R.A. and Dec. observations are in close agreement. The results have a high internal consistency.

3. The discussion of possible systematic error arising from differential atmospheric dispersion shows that atmospheric dispersion effects are bound up with the chromatic properties of the objectives and other factors. The effects of colour differences between Eros and the reference stars tend in consequence to assume a random nature. An investigation of the mean effect, which should depend on the distance of Eros, suggests that it is very small.

4. The constant of the lunar equation is 6″·4390 ± 0″·0015. The reciprocal of the mass of the Moon, inferred from this constant in conjunction with the solar parallax of 8″·790, is 81·271 ± 0·021.

5. A general adjustment of constants, by combining the results from the Eros observations with other relevant data, gives 9″·2266 ± 0″·0008 for the constant of nutation and 296·776 ± 0·054 for the reciprocal of the oblateness of the Earth's figure. On any reasonable assumption, the derived value of the constant of nutation is inconsistent with the adopted value of 9″·210. An accurate direct determination of the constant of nutation is much to be desired.

THE NON-RADIAL OSCILLATIONS OF POLYTROPIC STARS

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1. The problem of the non-radial oscillations of a fluid globe is of interest in the theory of the tidal distortion of binary stars. If the periods of rotation of the components differ from the orbital period, the tidal force produces a periodic disturbance at any point of either component. Provided that the period of this disturbance is long compared with the free periods of the stars, the tidal distortion differs little from that given by equilibrium theory. When, on the other hand, the period of the disturbance is comparable with the free periods, resonance is possible; this leads to an enhanced tidal distortion, resulting, in extreme cases, in the fission of a component.

The oscillations of a homogeneous incompressible liquid globe were considered by Kelvin.* The problem for a similar globe in rotation was considered by Bryan † and Love.‡ This problem is relatively tractable; but when a heterogeneous liquid globe is considered, the mathematical difficulties are much greater. For this reason, we consider here only the oscillations of non-rotating stars.

The oscillations of a heterogeneous compressible fluid sphere present a problem which is, in any case, of great mathematical complexity. As Pekeris has recently shown §, the problem is equivalent to the solution of two simultaneous second-order differential equations. Pekeris considered in detail the case of an initially homogeneous sphere of compressible fluid. This is clearly dynamically unstable, since, by transferring material from the outer layers to the centre, a mass of greater central density and smaller gravitational potential energy is obtained; and Pekeris' results indicated this instability.

Emden || considered approximately the oscillations of a polytropic gas sphere. His work was, however, vitiated by the approximations made in considering the equation of continuity, which he reduced to the form appropriate to a homogeneous incompressible fluid; the terms neglected and those retained are of the same order of magnitude. He obtained only one free period for oscillations in which the radial displacement is proportional to a surface harmonic of given order; there are, in fact, an infinite number of such periods.

In this paper we shall consider a polytropic sphere, whose specific heat may differ from that corresponding to convective equilibrium with the corresponding polytropic index. We shall show that oscillations of indefinitely large period are possible, and consider the bearing of this result on the tidal problem.

2. Consider a star of polytropic index n. In equilibrium the density ρ and pressure p at a distance r from the centre are connected with the central density ρ_c and pressure p_c by equations of the form

$$\rho = \rho_c y^n, \qquad p = p_c y^{1+n}, \tag{1}$$

where y satisfies Emden's equation

$$\frac{d}{dx}\left(x^2 \frac{dy}{dx}\right) = -x^2 y^n \tag{2}$$

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* *Collected Papers*, **3**, 384.
† *Phil. Mag.*, (5), **27**, 254, 1889.
‡ *Phil. Trans.*, (A), **153**, 187, 1889.
§ *Ap. J.*, **88**, 189, 1939.
|| *Gaskugeln*, p. 448.

and x is given by

$$r = r_0 x, \qquad r_0^2 = (1+n)p_c/4\pi G\rho_c^2 \tag{3}$$

(G is the gravitational constant).

In small oscillations, let the density and pressure, and the gravitational potential V at any point, be altered to $\rho+\delta\rho$, $p+\delta p$, $V+\delta V$, and let the (vector) displacement of material at any point from its equilibrium position be $\mathbf{h}$. Then, with the approximations usual in the theory of small oscillations, the equation of motion is

$$\rho\sigma^2\mathbf{h} = \text{grad}\,\delta p + \delta\rho\,\text{grad}\,V + \rho\,\text{grad}\,\delta V, \tag{4}$$

where $2\pi/\sigma$ is the period; and the equation of continuity is

$$\delta\rho = -\,\text{div}\,(\rho\mathbf{h}). \tag{5}$$

If the ratio of specific heats of the material is $1+1/N$, the relation between the changes in pressure and density of a given element of fluid is

$$\left(\delta p + R\frac{\partial p}{\partial r}\right)\Big/ p = \left(1+\frac{1}{N}\right)\left(\delta\rho + R\frac{\partial\rho}{\partial r}\right)\Big/\rho, \tag{6}$$

where R is the radial component of $\mathbf{h}$.

3. If the mass is strongly concentrated near the centre of the star, as is generally the case, the variations in the density do not produce great variations in the gravitational potential in the outer regions of the star. These regions have a considerable influence on the values of σ. We therefore ignore δV in deriving a first approximation to the periods.* The effect of this is to reduce the mathematical problem to the solution of a single differential equation of second order.

Eliminating $\mathbf{h}$ between (4) and (5), and remembering that grad $V = -(1/\rho)$ grad p, we now get

$$\sigma^2\delta\rho = -\nabla^2\delta p + \text{div}\left(\frac{\delta\rho}{\rho}\,\text{grad}\,p\right). \tag{7}$$

Also the radial component of (4) becomes

$$\rho\sigma^2 R = \frac{\partial\delta p}{\partial r} - \frac{\delta\rho}{\rho}\frac{\partial p}{\partial r}. \tag{8}$$

We shall consider oscillations in which each of $\delta\rho$, δp and R is the product of a surface harmonic of order s and a function of r. Then (7) becomes

$$\begin{aligned}\sigma^2\delta\rho &= -\frac{\partial}{r^2\partial r}\left(r^2\frac{\partial\delta p}{\partial r} - r^2\frac{\delta\rho}{\rho}\frac{\partial p}{\partial r}\right) + \frac{s(s+1)}{r^2}\delta p,\\ &= -\frac{\partial}{r^2\partial r}(\rho\sigma^2 r^2 R) + \frac{s(s+1)}{r^2}\delta p,\end{aligned} \tag{9}$$

by (8).

We introduce new non-dimensional quantities X, θ, ϕ, α, defined by

$$R = r_0 X, \qquad \delta p = p_c\theta, \qquad \delta\rho = \rho_c\phi, \qquad \sigma^2 = 4\pi G\rho_c\alpha/(1+n). \tag{10}$$

Using these definitions and (1) and (3), we can transform (6), (8) and (9) to the forms

$$y\phi(1+N) = N\theta - (n-N)Xy^n y', \tag{11}$$

$$y^n\alpha X = \frac{\partial\theta}{\partial x} - (1+n)\phi y', \tag{12}$$

$$\alpha\phi = -\frac{\partial}{x^2\partial x}(\alpha y^n x^2 X) + \frac{s(s+1)}{x^2}\theta. \tag{13}$$

* This was one of the approximations made by Emden.

Substituting for ϕ from (11) into (12) and (13), we get

$$\frac{\partial\theta}{\partial x}-\frac{(1+n)Ny'}{(1+N)y}\theta=X\left(y^n\alpha-\frac{(1+n)(n-N)}{1+N}y^{n-1}y'^2\right),$$

$$\frac{\partial}{x^2\partial x}(y^nx^2X)-\frac{n-N}{1+N}y^{n-1}y'X=\theta\left(\frac{s(s+1)}{\alpha x^2}-\frac{N}{(1+N)y}\right),$$

and if

$$z=Xy^{N(1+n)/(1+N)},\qquad w=\theta y^{-N(1+n)/(1+N)},\qquad Q=-n+\frac{2N(1+n)}{1+N},\tag{14}$$

these can be reduced to

$$\frac{\partial w}{\partial x}=zy^{-Q}\left(\alpha-\frac{(1+n)(n-N)}{1+N}\frac{y'^2}{y}\right),\tag{15}$$

$$\frac{\partial}{\partial x}(x^2z)=wy^Q\left(\frac{s(s+1)}{\alpha}-\frac{Nx^2}{(1+N)y}\right),\tag{16}$$

We require solutions of these equations which make $R=0$ at the centre and $\delta p=0$ at the boundary of the stars. These conditions are equivalent to $z=0$ at $x=0$, and $w=0$ at the boundary. It is possible to satisfy these conditions only for a certain set of values $\alpha_1, \alpha_2, \ldots$ of α; the corresponding values of $\mathbf{h}$, etc., are denoted by $\mathbf{h}_1, \mathbf{h}_2, \ldots$, etc.

4. Equations (15) and (16) are equivalent to a single second-order equation in the variable w or z. If α is very large,

$$\left(1+\frac{1}{N}\right)\frac{\partial}{\partial x}\left(y^{1-Q}\frac{\partial}{x^2\partial x}(x^2z)\right)\doteq-\alpha zy^{-Q},$$

and if α is very small,

$$\frac{1+N}{(1+n)(n-N)}\frac{\partial}{\partial x}\left(\frac{x^2y^{1+Q}}{y'^2}\frac{\partial w}{\partial x}\right)\doteq-\frac{s(s+1)}{\alpha}wy^Q.$$

Each of these is of the Sturm-Liouville type; the first has an infinite number of solutions with α indefinitely large, the second an infinite number with α indefinitely small. Hence an infinite number of oscillations with very long periods are possible, as well as an infinite number with indefinitely small periods.

In the oscillations of very short period δp is large, and the motion, which is chiefly radial, is largely actuated by the pressure variations. In those of very long period δp and $\delta\rho$ are small, $\delta\rho$ being the larger of the two unless $n-N$ is small; the motion is chiefly horizontal and is due to the action of gravity in attempting to smooth out the density differences on any sphere concentric with the star. In the cases which have been studied in detail there is a single oscillation of intermediate period such that R and $-\delta\rho$ have the same sign at all points of any assigned radius; this may be called the fundamental (*f*-) oscillation. The oscillations of greater and less period may be called respectively gravitational (*g*-) and pressure (*p*-) oscillations. The *g*- and *p*-oscillations of high order (those whose periods are very different from that of the *f*-oscillation) possess a large number of nodes and loops (spheres concentric with the star on which $R=0$ and $\delta p=0$ respectively). The node of least radius is nearer the centre than the loop of least radius for the *p*-oscillations, and *vice versa* for the *g*-oscillations.

The free periods all decrease if either n increases (greater central condensation) or N decreases (smaller compressibility). If $n=N$ the periods of all the *g*-oscillations are infinite; the equilibrium of the star is then clearly neutral for indefinitely slow internal displacements of the material. When $n<N$, the star is unstable. If $N=0$ (incompressible fluid), the periods of all the *p*-oscillations vanish. The variations in density,

$\delta\rho$, are very small for the g-oscillations when $n - N$ is small, and for the p-oscillations when N is small.

5. Equations (15) and (16) are of a form suitable for numerical integration, and after a number of trial solutions a fairly accurate value of α can be obtained. We can use the corresponding values of w and z to correct the error involved in the neglect of δV, using the usual methods of perturbation theory. Suppose that the retention of δV in (4) results in an increase in the value $\mathbf{h}_m$ of $\mathbf{h}$ corresponding to $\alpha = \alpha_m$ of amount $\Sigma a_l \mathbf{h}_l$, where the quantities a_l are so small that their squares and products can be neglected.* Then, by (5) and (6), $\delta\rho_m$ and δp_m are increased by $\Sigma a_l \delta\rho_l$ and $\Sigma a_l \delta p_l$, and (4) can be written

$$\rho\sigma^2(\mathbf{h}_m + \Sigma a_l \mathbf{h}_l) = \text{grad}\,(\delta p_m + \Sigma a_l \delta p_l) + (\delta\rho_m + \Sigma a_l \delta\rho_l)\,\text{grad}\,V + \rho\,\text{grad}\,\delta V,$$

or, using the unperturbed equation and neglecting second-order quantities,

$$\rho(\sigma^2 - \sigma_m{}^2)\mathbf{h}_m = \rho\Sigma a_l(\sigma_l{}^2 - \sigma_m{}^2)\mathbf{h}_l + \rho\,\text{grad}\,\delta V.$$

Take the scalar product of this into $\mathbf{h}_m$ and integrate throughout the volume τ of the star. The orthogonality condition satisfied by the eigen-functions $\mathbf{h}_m$ is of the form †

$$\int \rho \mathbf{h}_l \,.\, \mathbf{h}_m d\tau = 0 \qquad (l \neq m), \tag{17}$$

where the notation $\mathbf{a} \,.\, \mathbf{b}$ is used for the scalar product of the vectors $\mathbf{a}$, $\mathbf{b}$. Hence

$$\begin{aligned}(\sigma^2 - \sigma_m{}^2)\int \rho \mathbf{h}_m \,.\, \mathbf{h}_m d\tau &= \int \rho \mathbf{h}_m \,.\, \text{grad}\,\delta V d\tau \\ &= \int \delta\rho_m \delta V d\tau \end{aligned} \tag{18}$$

using an integration by parts and substituting from (5). This equation gives the correction to the period $2\pi/\sigma_m$.

The value of δV is found from the Poisson equation

$$4\pi G\delta\rho_m = \nabla^2\delta V = \frac{\partial}{r^2\partial r}\left(r^2\frac{\partial\delta V}{\partial r}\right) - \frac{s(s+1)}{r^2}\delta V.$$

The solution of this is

$$\delta V = -\frac{4\pi G}{2s+1}\left\{r^s\int_r^\infty \delta\rho_m r^{1-s}dr + r^{-1-s}\int_0^r \delta\rho_m r^{s+2}dr\right\}, \tag{19}$$

the integration being along the radius.

In terms of the non-dimensional variables y, z, w and x, equations (18) and (19) can be written

$$(\alpha - \alpha_m)\int_0^\infty \left(y^{-Q}x^2z^2 + \frac{s(s+1)}{\alpha^2}y^Q w^2\right)dx = \int_0^\infty \phi x^2 v dx, \tag{20}$$

where

$$v = \frac{\rho_c}{p_c}\delta V = -\frac{1+n}{2s+1}\left\{x^s\int_x^\infty \phi x^{1-s}dx + x^{-1-s}\int_0^x \phi x^{s+2}dx\right\}. \tag{21}$$

It is clear from (18) and (19) that the errors in period due to the neglect of δV are small for higher oscillations of both p- and g-types, because of the large number of changes of sign of $\delta\rho_m$ on any radius. Especially is this true for the higher g-oscillations, since for these $\delta\rho_m$ is small in absolute value. Also, as (19) shows, the error is small when s is large.

* The time-factor in $\mathbf{h}_l$ is supposed omitted.

† For a system with a finite number of degrees of freedom whose kinetic energy is $\frac{1}{2}\Sigma\Sigma a_{\alpha\beta}\dot{q}_\alpha\dot{q}_\beta$, the orthogonality condition is of the form $\Sigma\Sigma a_{\alpha\beta}q_\alpha{}^{(l)}q_\beta{}^{(m)} = 0$ $(l \neq m)$ (see Lamb, *Higher Mechanics*, 2nd ed., p. 228). Equation (17) is the generalisation of this to a continuous system.

6. The case $s=2$ is the important one for the tidal problem, and this alone has been considered numerically. The periods of the f-oscillation and the g-oscillation of largest period ($g1$) have been found for a polytrope of index 3 in the cases (i) $N=\frac{3}{2}$, corresponding to a monatomic gas, (ii) $n-N$ very small and positive, corresponding to a star nearly in neutral equilibrium. The first approximations obtained by solving (15) and (16), and the corrections given by (18), are listed below: M denotes the total mass, r_1 the radius of the star. For comparison, the results for the f-oscillation of a homogeneous incompressible star ($n=0=N$) are also given.

Oscillation	α_m	$\alpha-\alpha_m$	α	σ^2
$N=\frac{3}{2}$, f	0·2325	−0·0186	0·2139	$8{\cdot}69GM/r_1^3$
$N=\frac{3}{2}$, $g1$	0·1308	−0·0142	0·1166	$4{\cdot}72GM/r_1^3$
$n-N$ small, f	0·1583	−0·0099	0·1484	$6{\cdot}03GM/r_1^3$
$n-N$ small, $g1$	$0{\cdot}06(n-N)$	$O(n-N)^2$	$0{\cdot}06(n-N)$	$2{\cdot}44(n-N)GM/r_1^3$
$n=0=N$, f	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{4}{15}$*	$\frac{4}{5}GM/r_1^3$*

It is clear from this table that the first approximations to α found by solving (15) and (16) are not greatly in error for a polytrope of index 3. The values of $\mathfrak{h}$, $\delta\rho$, etc., obtained from this approximation are not very accurate near the centre of the star, but are not greatly in error at distances from the centre greater than half the radius.

The value of σ^2 given by Emden was $2GM/r_1^3$. This is much smaller than the true values for the f-oscillation.

7. It remains to consider the bearing of these results on the tidal problem in double stars. Since the components of a double star rotate, their free periods will differ somewhat from those given above, the longest periods being most affected: but the general conclusion that indefinitely long periods are possible is still valid.† We cannot, therefore, rule out the possibility of resonance, however long may be the period of the tidal disturbance at any point of the star. But reasons may be urged why this should not necessarily produce important effects in actual stars.

(A) The oscillation with which there is resonance contributes relatively little to the equilibrium distortion of the star, and so can be responsible for a greatly enhanced total tide only if there is very close resonance.

(B) In the oscillation in question, the fluid undergoes considerable horizontal displacements even when the density-variations are small. Thus, when the oscillation is highly stimulated by resonance, second-order effects will appear, which may alter the free period so that there is no longer exact resonance. In this case the increase in amplitude of the displacements will be checked before the density-changes and the consequent distortion of the surface become large.

We consider (B) first. The period of the tidal disturbance at any point of the star is $\pi/(\omega-\omega_1)$, where ω is the orbital angular velocity and ω_1 the angular velocity of the star. This is not likely to be much less than $2\pi/\omega$, and ω^2 can hardly exceed $GM/4r_1^3$, its value for two equal stars in contact. Comparing this with the values of σ^2 given above, we see that the tidal disturbance can only be in resonance with a g-oscillation, which will be one of high order unless $n-N$ is small. In a high-order g-oscillation, $\delta\rho$ is small unless the horizontal displacements are considerable; the same is true of all the g-oscillations if $n-N$ is small. Argument (B) follows.

To discuss (A), suppose the star is subjected to a (small) periodic tidal force of potential $\Omega\cos\lambda t$. Using an argument similar to that of § 5, we can show that the

* These are exact values for α and σ^2.

† The general effect of the rotation is that, corresponding to each free period of a non-rotating star, there are several free periods of the rotating star, some greater, some smaller than the original period. The character of the corresponding motions is also altered: for large periods, the horizontal motions are nearly orthogonal to the horizontal pressure gredients.

resulting displacement at any point can be expressed in terms of the eigen-functions $\mathbf{h}_m$ by a series $\Sigma b_m \mathbf{h}_m \cos \lambda t$, where

$$b_m(\sigma_m{}^2 - \lambda^2)\int \rho \mathbf{h}_m \cdot \mathbf{h}_m d\tau = \int \delta\rho_m \Omega d\tau.$$

The resulting change in potential energy of the star is

$$\cos^2 \lambda t \int \Sigma b_m \delta\rho_m \Omega d\tau = \cos^2 \lambda t \Sigma \frac{c_m}{1 - \lambda^2/\sigma_m{}^2},$$

where

$$c_m = \left(\int \delta\rho_m \Omega d\tau\right)^2 \Big/ \sigma_m{}^2 \int \rho \mathbf{h}_m \cdot \mathbf{h}_m d\tau. \tag{22}$$

Thus in the equilibrium tide ($\lambda = 0$) the part of the change in potential energy due to coupling with the oscillation of period $2\pi/\sigma_m$ is $c_m \cos^2 \lambda t$; this is increased in the ratio $1/(1 - \lambda^2/\sigma_m{}^2)$ in a periodic tide.

For the case $n = 3$, $N = \frac{3}{2}$, the coefficient c_m for the g_1 oscillation is only one-fifth as large as for the f-oscillation. For the higher g-oscillations c_m is much smaller, since $\delta\rho_m$ changes sign repeatedly on any radius. Smaller values of c_m are also obtained for the lower g-oscillations when $n - N$ is small, since then $\delta\rho_m$ is small for these. Argument (A) follows.

Thus, in spite of the possibility of resonance, the tidal distortion of double stars may be expected not to differ seriously from the equilibrium distortion, save, perhaps, in rare cases. There seems to be no serious likelihood that tidal disruption of a star will take place through resonance.

8. Recently the asymmetry of light-curves of eclipsing variables has been studied by P. H. Taylor.* He found that for stars exhibiting a marked asymmetry, approximately

$$P/\bar{\rho}^{\frac{1}{2}} = \text{const.}, \tag{23}$$

where P is the orbital period and $\bar{\rho}$ the mean density of the component of lower density. Now, the free periods of stars built on a similar model satisfy a homology relation of the form (23). He therefore suggested that the asymmetry was due to resonance between a free oscillation of this component and the periodic variation of the tidal force on it, due to the orbital variation of the distance from its companion. If this interpretation is correct, resonance is more important than the above arguments would suggest.

The interpretation seems, however, to be questionable. It rests on the assumption that the stars have a single free period with which there is resonance. In fact, however, in the range in question (which is that of the higher g-oscillations) the free periods lie fairly close together, and, for resonance, $P/\bar{\rho}^{\frac{1}{2}}$ can take a whole series of values not very widely separated. Thus, allowing for observational errors, no actual relationship between P and $\bar{\rho}$ would be observed.

An explanation of the existence of asymmetry can be given on the following lines. It is known that the variations in the light emitted by a star pulsating radially lag behind the variations in radius.† When the orbital and rotational angular velocities ω, ω_1 are different, the tidal distortion of a component is a forced oscillation; we may expect that the light variations at any point in such a forced oscillation will also lag behind the vertical displacement. If so, the observed light variations will show asymmetry whose sign will depend on the sign of $\omega - \omega_1$. A relation of the form (23) will be found if the lag of the light variations is enhanced for very close pairs.

9. Our results can also be related to the resonance theory of the origin of the Moon, studied by Darwin and Jeffreys, among others. Jeffreys‡ considered (in a two-

* *Ap. J.*, **94**, 46, 1941.
† See, especially, Eddington, *M.N.*, **101**, 182, 1941.
‡ *M.N.*, **78**, 116, 1917.

dimensional model) the free periods of oscillation of the primitive Earth, supposed to consist of a core of heavy liquid and an outer layer of lighter liquid. He found free periods considerably larger than those of a homogeneous mass of the same radius and mean density, so that resonance between the solar tide and the corresponding oscillations was possible. By a generalisation of the theory given above, the same result would be obtained if the Earth were supposed to consist of a compressible liquid whose density varied continuously with depth, provided that the mass was stable for internal displacements (corresponding to the condition $n > N$ above).

In a later paper* Jeffreys concluded that frictional forces due to turbulance at the surface of the core were so large that the tidal distortion could not be amplified by resonance up to the point where fission occurs. This argument need not apply, however, if the Earth's density varies continuously, since the stability of the mass for internal displacements prevents turbulence from appearing until large velocities of streaming are present.

The arguments (A) and (B) given above (§ 7) can be urged in this connection, but they have less weight for the primitive Earth than for gaseous stars. The greater uniformity of density in the Earth results in an increase in all the free periods. In consequence, resonance can occur with a g-oscillation not of high order; the oscillation with which Jeffreys found resonance corresponds roughly to our g_1 oscillation. Such an oscillation contributes more to the total tide than a high-order g-oscillation, and the horizontal motions corresponding to a given surface distortion are smaller. Thus, while second-order effects must be taken into account, they do not wholly rule out the possibility of fission. It would be misleading to assert that these considerations clear up the difficulties of the resonance theory of the Moon's origin; but they suggest that further work on the problem is desirable.

Summary

The non-radial oscillations of a polytropic gas sphere are discussed. Periods are obtained numerically in certain special cases by an approximate method, and the error of the approximation is estimated. An infinite number of indefinitely long periods is shown in general to exist. The possibility of resonance between long-period oscillations and the tidal force in double stars is considered, and it is concluded that such resonance will not in general greatly affect the tidal distortion. The bearing of the results on the resonance theory of the Moon is discussed.

Appendix.—Tables giving the variation of $X(=R/r_0)$, $\theta(=\delta p/p_c)$, and $\phi(=\delta\rho/\rho_c)$ along any given radius. The constant of proportionality in the solution is adjusted to make X tend to equal $x(=r/r_0)$ as x becomes small. In the numerical work, the British Association tables of Emden functions were used.

* *M.N.*, 91, 169, 1930.

[TABLES.

$n=3, \quad N=\frac{3}{2}$

	f-oscillation			*g*1-oscillation		
x	X	$10^3\theta$	$10^2\phi$	X	$10^3\theta$	$10^3\phi$
0·2	0·1982	4·442	1·040	0·1951	2·441	6·137
0·4	0·3851	15·45	3·737	0·3633	7·889	21·44
0·6	0·5546	28·07	7·115	0·4843	12·34	38·68
0·8	0·6981	36·97	10·040	0·5504	12·95	51·12
1·0	0·8168	39·97	11·845	0·5661	9·647	56·37
1·2	0·9124	37·29	12·335	0·5412	4·359	53·46
1·4	1·001	31·23	11·882	0·4911	−0·850	46·59
1·6	1·068	24·09	10·642	0·4253	−4·708	38·05
1·8	1·143	17·28	9·196	0·3505	−6·880	29·69
2·0	1·223	11·64	7·713	0·2736	−7·506	22·31
2·2	1·311	7·341	6·343	0·2017	−7·120	16·49
2·4	1·420	4·763	5·220	0·1330	−6·246	12·09
2·6	1·551	3·031	4·285	0·0691	−5·168	8·790
2·8	1·713	2·072	3·561	0·0114	−4·132	6·452
3·0	1·914	1·611	2·995	−0·0442	−3·223	4·769
3·2	2·162	1·444	2·571	−0·0940	−2·469	3·572
3·4	2·473	1·440	2·255	−0·1430	−1·873	2·722
3·6	2·856	1·509	2·017	−0·1913	−1·407	2·106
3·8	3·327	1·581	1·840	−0·2401	−1·051	1·655
4·0	3·908	1·629	1·700	−0·2888	−0·7807	1·327
4·2	4·618	1·640	1·590	−0·3383	−0·5772	1·082
4·4	5·487	1·597	1·492	−0·3938	−0·4239	0·8878
4·6	6·549	1·508	1·402	−0·4525	−0·3091	0·7362
4·8	7·845	1·375	1·309	−0·5168	−0·2224	0·6105
5·0	9·426	1·208	1·211	−0·5870	−0·1575	0·5059
5·2	11·34	1·021	1·102	−0·6664	−0·1089	0·4153
5·4	13·68	0·8198	0·979	−0·7523	−0·0730	0·3355
5·6	16·59	0·6210	0·844	−0·8487	−0·0468	0·2641
5·8	19·99	0·4363	0·695	−0·9579	−0·0281	0·1994
6·0	23·96	0·2757	0·535	−1·078	−0·0154	0·1418
6·2	28·82	0·1488	0·362	−1·222	−0·0073	0·0912
6·4	34·42	0·0621	0·221	−1·397	−0·0027	0·0494
6·6	41·52	0·0152	0·091	−1·552	−0·0006	0·0189
6·8	51·41	0·0006	0·010	−1·670	−0·0000	0·0022

$n=3$, $n-N$ small

	f-oscillation			*g*1-oscillation		
x	X	$10^2\theta$	$10^2\phi$	$10X$	$10^3\theta/(n-N)$	$10^3\phi/(n-N)$
0·2	0·2022	0·3119	0·2355	1·964	6·723	8·268
0·4	0·4178	1·196	0·9204	3·686	21·92	27·74
0·6	0·6601	2·504	1·991	5·009	35·03	47·92
0·8	0·9461	4·060	3·373	5·861	38·06	58·13
1·0	1·289	5·646	4·952	6·275	30·16	55·38
1·2	1·713	7·114	6·646	6·311	15·82	42·29
1·4	2·244	8·341	8·370	6·080	0·96	24·50
1·6	2·913	9·292	10·08	5·671	−10·61	7·41
1·8	3·745	9·938	11·71	5·178	−17·48	− 6·31
2·0	4·808	10·312	13·26	4·650	−20·11	−15·54
2·2	6·143	10·432	14·71	4·142	−19·66	−20·48
2·4	7·805	10·348	16·03	3·653	−17·44	−22·52
2·6	9·890	10·085	17·22	3·203	−14·97	−22·22
2·8	12·49	9·703	18·30	2·802	−12·13	−20·68
3·0	15·71	9·215	19·24	2·444	− 9·473	−18·32
3·2	19·69	8·652	20·03	2·144	− 7·246	−15·82
3·4	24·61	8·035	20·67	1·871	− 5·436	−13·37
3·6	30·63	7·375	21·13	1·633	− 4·011	−11·10
3·8	38·01	6·691	21·41	1·420	− 2·917	− 9·078
4·0	47·01	5·995	21·48	1·241	− 2·091	− 7·333
4·2	57·95	5·293	21·31	1·078	− 1·481	− 5·860
4·4	71·20	4·598	20·89	0·9393	− 1·033	− 4·628
4·6	87·20	3·922	20·21	0·8123	− 0·7087	− 3·613
4·8	106·5	3·271	19·24	0·6964	− 0·4761	− 2·776
5·0	129·7	2·657	17·98	0·6027	− 0·3125	− 2·100
5·2	157·6	2·087	16·41	0·5113	− 0·1992	− 1·557
5·4	191·1	1·575	14·56	0·4217	− 0·1222	− 1·126
5·6	230·9	1·127	12·47	0·3467	− 0·0713	− 0·787
5·8	278·3	0·7498	10·16	0·2806	− 0·0389	− 0·525
6·0	334·1	0·4511	7·736	0·2192	− 0·0192	− 0·329
6·2	401·0	0·2331	5·316	0·1629	− 0·0082	− 0·186
6·4	479·5	0·1133	3·074	0·1116	− 0·0027	− 0·089
6·6	567	0·0219	1·248	0·0644	− 0·0005	− 0·030
6·8	675	0·0008	0·152	0·0209	− 0·0000	− 0·003

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ON THE RADIAL VELOCITY CURVE OF δ CEPHEI

H. A. Brück and H. E. Green

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Investigations of radial velocity curves of Cepheids have been published by Rufus * and other Michigan observers † which indicate the existence of marked relative displacements between spectral lines originating at different levels in the stellar atmosphere. These levels have been taken as equivalent to the heights of the same lines in the solar chromosphere.‡ The Michigan observers divided the lines into different groups according to their chromospheric heights and derived separate radial velocity curves for the various groups. Such curves were found to differ considerably in shape, amplitude and phase. The individual results were conflicting in that three out of five Cepheids seemed to reveal an increase of amplitude with height, apart from minor differences between the velocity curves of different levels, whilst the two others showed just the opposite or no such effect. Velocity curves from high-level lines seemed to exhibit in all cases a lag of phase compared with those derived from lower-level lines.

These and other results of the Michigan observations are based on spectrograms secured with a single-prism spectrograph giving a dispersion of 40 A./mm. near $H\gamma$. With such dispersion the vast majority of lines in spectra of normal Cepheid variables must be unresolved blends composed of several contributors whose relative intensity may vary considerably during the cycle of the variable. The centres of blends may therefore show apparent shifts with phase which, being different for different blends, may result in marked relative displacements like those which are stated to exist between lines of different levels.

It should be mentioned here that Jacobsen § using six three-prism plates of δ Cephei did not find any systematic changes between radial velocity curves obtained from lines at different heights. The same negative result, *i.e.* no convincing evidence for the existence of relative displacements, was found by Mendenhall ‖ in a study of the variable β Cephei, based on three-prism spectrograms.

About one hundred single-prism spectrograms of δ Cephei have been discussed by Petrie.¶ As these plates had been obtained with the same instrument used for the earlier Michigan observations, it is of particular interest that Petrie came to the conclusion that there was no definite indication of velocity differences and that an apparent phase effect between hydrogen and other lines was only of the same order as the errors of observation.

At Cambridge a number of four-prism spectrograms of δ Cephei was available, which had been secured with the Newall 25-inch refractor of the Solar Physics Observatory and had originally been used for a spectrophotometric study of line intensities and their variations during the cycle.** These spectrograms have a dispersion of 13 A./mm. near $H\gamma$, and it seemed worth while to make use of them for an additional attempt to test the existence of relative displacements.

The spectrograms are in good focus over a wave-length region extending from

* *Proc. Nat. Acad.*, 10, 264, 1924; *Pub. Obs. Michigan*, 4, 101, 1932; *ibid.*, 4, 151, 1932.

† R. H. Curtiss, *P.A.S.P.*, 38, 148, 1926; D. W. Lee, *Pub. Obs. Michigan*, 4, 109, 1932; J. A. Aldrich, *ibid.*, 4, 75, 1932; R. M. Petrie, *ibid.*, 5, 9, 1932. Compare also F. Henroteau, *Pub. Dom. Obs. Ottawa*, 9, 105, 1925, and 9, 127, 1928; R. F. Sanford, *Ap. J.*, 66, 170, 1927.

‡ See S. A. Mitchell, *Ap. J.*, 71, 1, 1930.

§ *Lick Obs. Bull.*, 12, 138, 1926.

‖ *Lick Obs. Bull.*, 14, 133, 1930.

¶ *Pub. Obs. Michigan*, 6, 37, 1934.

** H. A. Brück, Cambridge Thesis, 1940.

λ 4250 to λ 4650. In the course of the spectrophotometric analysis about 200 lines had been studied within this region of which 10 per cent. were found to be unblended on the Cambridge spectrograms or to be blends with one strongly predominant contributor. Out of these lines, seventeen have been selected for the present determination of radial velocities. They are given in Table I, together with their identifications according to Krieger * and their chromospheric heights taken from Mitchell's tables (*loc. cit.*).

With the exception of λ 4274·8 of neutral chromium, λ 4425·6 of neutral calcium and λ 4554·1 of ionised barium, all lines belong to neutral or ionised iron and titanium. Their chromospheric heights vary between 450 and 2500 km. with nine lines lying under 1000 km., eight at and above 1000 km.

TABLE I

Selected Lines

Wave-length	El.	E.P. volts	Height km.	Wave-length	El.	E.P. volts	Height km.
4274·8	*Cr* I	0·00	1500	4501·4	*Ti* II	1·11	2500
4312·9	*Ti* II	1·18	1200	4508·3	*Fe* II	2·84	900
4383·5	*Fe* I	1·48	1600	4515·3	*Fe* II	2·83	800
4386·9	*Ti* II	2·59	450	4520·2	*Fe* II	2·80	800
4404·8	*Fe* I	1·55	1200	4541·5	*Fe* II	2·84	700
4425·6	*Ca* I	1·87	600	4554·1	*Ba* II	0·00	2000
4450·5	*Ti* II	1·08	1000	4563·8	*Ti* II	1·22	2500
4466·6	*Fe* I	2·82	500	4576·4	*Fe* II	2·83	500
4476·0	*Fe* I	2·83	500				

Most of the spectrograms were measured twice independently, and the positions of these seventeen lines determined in the usual manner. Their displacements were referred to their positions at phase $2^{d}.27$, represented by a spectrogram of excellent quality, when the radial velocity of δ Cephei should be very nearly midway between its maximum and minimum values. Measures of two plates were combined as means leading to fourteen normal places for the velocity curve. When all lines are taken together and are given equal weights, the mean radial velocity curve shown in fig. 1 is derived. Numerical values of the radial velocities are given in the second column of Table II.

TABLE II

Radial Velocities (km./sec.)

Phase	All Lines	Low Level	Int. Level	*Fe* I	*Fe* II	*Ti* II
d						
0·00	−19·2	−19·0	−19·5	−20·3	−18·0	−18·4
0·24	−16·4	16·5	16·3	16·5	15·5	−17·2
0·42	−14·8	13·1	16·8	15·3	11·6	−16·6
0·60	−12·5	12·6	12·4	13·8	11·4	−11·4
0·83	−12·2	14·0	10·2	14·0	12·6	− 9·8
1·34	− 7·4	9·3	5·3	8·5	8·5	− 6·0
1·80	− 1·7	1·8	− 1·6	− 0·8	2·0	− 0·5
2·22	− 0·9	− 2·1	+ 0·4	+ 0·7	− 2·7	+ 0·6
2·67	+ 5·9	+ 5·4	4·7	6·7	+ 5·4	+ 5·8
3·37	+13·1	15·2	11·7	12·7	...	+11·1
4·16	+19·7	21·4	15·9	19·4	21·8	+16·6
4·54	+ 7·6	+ 7·0	+ 8·2	+ 8·6	+ 7·1	+ 6·5
4·78	− 5·2	− 6·0	− 4·3	− 4·6	− 7·5	− 4·0
5·05	−10·8	− 9·2	−12·8	−11·6	−10·6	−11·9

* *Ap. J.*, 79, 98, 1934. Compare also A. Pannekoek and G. B. van Albada, *Pub. Astron. Inst. Amsterdam*, No. 6, 1939.

Phases are taken from the Babelsberg ephemeris * for the times of light maximum of δ Cephei. A photoelectric light-curve of the variable as observed by Smart † is given for comparison. The observations of radial velocity are not equally distributed over the cycle and there are rather too few observations near minimum phase when it proved difficult to obtain spectrograms of the same quality as at other phases. Both spectrograms which furnish the data for phase $3^d{\cdot}37$ are under-exposed towards long wave-lengths so that only the first ten lines of Table I have been used for the determination of the radial velocity at this phase. If the deviations of the velocities of individual lines

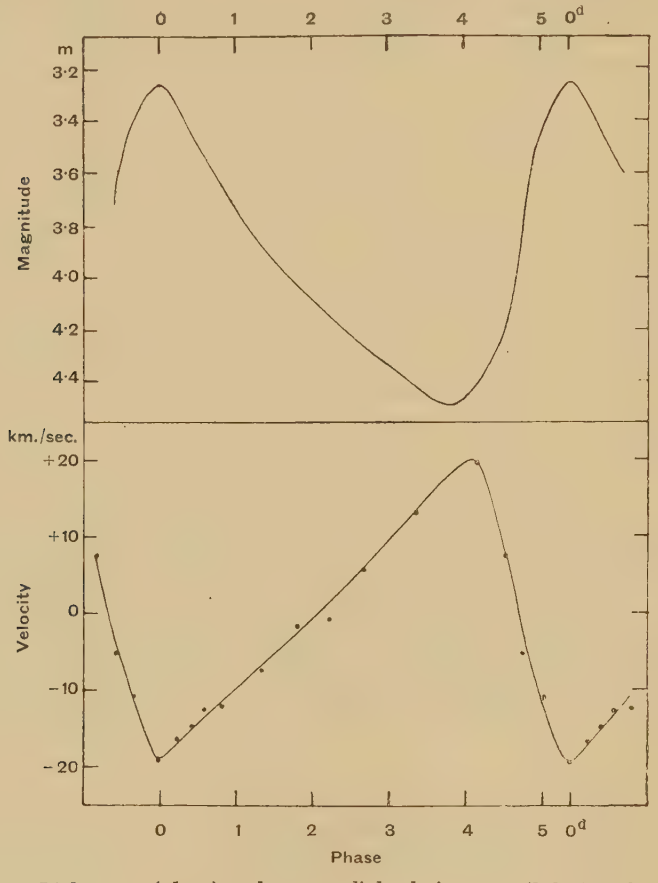

FIG. 1.—*Light curve (above) and mean radial velocity curve (below) of δ Cephei.*

from the mean values are attributed to accidental errors, the probable error of the mean radial velocities is found to vary between ±0·4 and ±0·6 km./sec. for the different phases.

In order to test the existence of level effects, the material has been separated into two groups of lines below and above the height of 1000 km. The first group contains nine lines with an average height of 600 km. corresponding to the "low level" of the Michigan observations. There are eight lines in the second group with an average height of 1700 km. in correspondence with Petrie's "intermediate level." Separate radial velocity curves from the lines in these two groups are given in Table II, whereas their differences in the sense "intermediate minus low level" are plotted against phase in fig. 2 (A).‡

* *Kleinere Veröffentl. Universitäts Sternwarte Berlin-Babelsberg. Katalog und Ephemeriden Veränderlicher Sterne.* Published annually. † *M.N.*, 95, 644, 1935.

‡ In figs. 2 and 3 ordinates are drawn at twice the scale of that in fig. 1.

Considering the errors of observation, both levels give essentially the same velocity curve. It might appear as if the broken curve would fit the observations better than a straight line, but there is no clear variation of sign of the residuals with phase, and the standard deviation of the individual points is only insignificantly larger for a straight line than for the broken curve ($\pm 2{\cdot}8$ and $\pm 2{\cdot}3$ km./sec. respectively). Our conclusion is that our four-prism spectrograms show no evidence of the existence of relative displacements between lines of intermediate and low atmospheric level.

Of high-level lines, $H\gamma$ (chromospheric height 8000 km.) is present on the Cambridge plates, but could not be measured with equal accuracy and has therefore not been

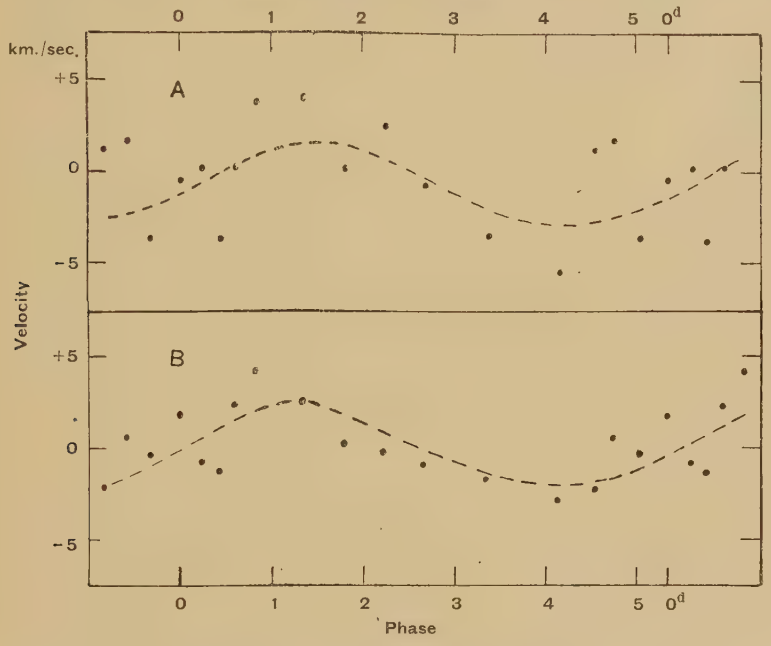

FIG. 2.—*Radial velocity differences between different groups of lines.*

(A) *Lines of intermediate level minus lines of low level.*
(B) *Lines of ionised titanium minus lines of neutral iron.*

included among the lines of Table I. Velocity differences between $H\gamma$ and lines of low level or all lines together, if plotted against phase, can be well represented by a straight line round which the individual observations scatter with a standard deviation of as much as $\pm 3{\cdot}5$ km./sec. As far as can be concluded from the limited accuracy of the measurements of $H\gamma$, no systematic difference is found between the velocity curve of this high-level line and that of the low-level group.

An attempt has also been made to compare the radial velocities of two groups of neutral iron and ionised titanium lines. Among the four iron lines represented in Table I no systematic difference was found between the velocities of the two lines with an excitation potential of 2·82 and the two others with a mean E.P. of 1·51 volts. These four lines were therefore considered together. Velocities derived from this group of neutral iron lines and from a second group of four ionised titanium lines (mean E.P. 1·15 volts) are given in Table II and their differences are shown in fig. 2 (B).

The broken curve which fits the observations best is similar to that of fig. 2 (A). In both cases the amplitude is about 4·5 km./sec., while in fig. 2 (B) the standard deviation of a point from the curve is $\pm 1{\cdot}4$ km./sec. against $\pm 2{\cdot}3$ km./sec. in fig. 2 (A). There appears to be some indication of small shifts between the groups of neutral iron and

ionised titanium lines which, however, would have to be confirmed by more observations on high dispersion spectrograms before it can be taken to be real.

For the group of five lines of ionised iron in Table I whose velocities are included among the data of Table II, no observations are available at the important phase $3^d{\cdot}37$, and while their velocity differences against the group of neutral iron lines could possibly be represented by a curve similar to that in fig. 2 (B), no definite conclusion can be obtained in this case in which the observations show also considerable scatter.

While for the groups of lines which have been studied so far only doubtful relative displacements, if any, are observed, at least one individual line seems definitely to show shifts. This is λ 4425·6 of neutral calcium whose residual radial velocities are plotted against phase in fig. 3 (A). Near light minimum this line appears to be shifted by an

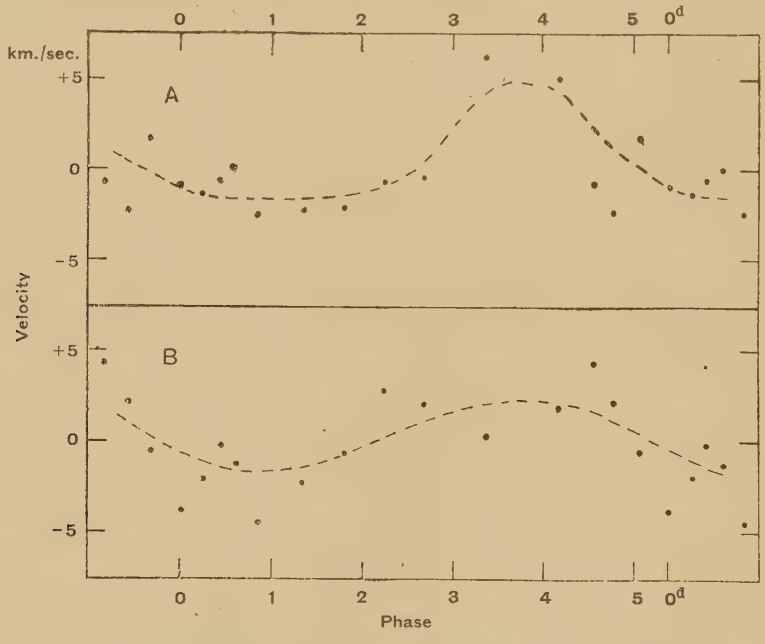

FIG. 3.—*Residual radial velocities of* (A) λ 4425·6 *of neutral calcium, and* (B) λ 4476·0 *of neutral iron.*

amount of about +6 km./sec. from its position at light maximum. Since the hump in the broken curve of fig. 3 (A) depends on the measurements of only four spectrograms, the exact value of the shift cannot be determined, but its existence appears to be established and has been checked by several independent measurements of the spectrograms concerned. The line is well resolved on the Cambridge spectrograms so that the effect cannot be explained by blends.

A similar effect may perhaps be indicated by the residual velocities of λ 4476·0 of neutral iron (fig. 3 (B)). In the cases of all other lines, residuals if plotted against phase are more or less scattered round straight lines with the average deviations of the velocities at different phases varying from line to line between ±2·5 and ±3·0 km./sec.

Summary

Cambridge four-prism spectrograms of δ Cephei do not indicate any definite relative displacements between lines of different atmospheric level, confirming thereby earlier negative results of Petrie and Jacobsen. There may exist small differences between the velocity curves of groups of neutral iron and ionised titanium lines, and there is evidence for the existence of relative shifts in the case of the neutral calcium line λ 4425·6.

GALACTIC ABSORPTION AND APPARENT DISTRIBUTION OF SPECTRAL TYPES

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(Communicated by the Director, Norman Lockyer Observatory)

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1. The question of galactic absorption offers more difficulties than can be solved at present. The only reliable methods for determining whether or not an absorbing stratum is present require a knowledge of stellar spectra throughout the galaxy, and are therefore restricted to a relatively small volume because such knowledge is restricted, by the optical power of the instruments, to a limited range of apparent magnitudes. It is, however, possible to extend this range considerably by using colour indices (CI) instead of the spectral types and thus making much greater distances available for investigation.

In the present paper it is shown that selective absorption has an effect on the apparent spatial distribution of spectral types, and an attempt is made to suggest a method of approach from this viewpoint. Some such method is necessary, as pointed out by Bok *, "before it will be possible to make full use of colour-index surveys in studies of galactic structure." It may also be important for other astrophysical questions where, at present, "investigations based on colour indices must be treated with reserve," as mentioned by Greaves.†

A recent paper ‡ (designated later as G I) investigated the influence of a continuous and uniform absorbing layer in the galactic plane and the relation between true star distances and those determined by a falsified CI, and of the probable limits of the absorption coefficients of such a layer. The influence was shown to be dependent on a parameter b the qualities of which will be discussed in the present paper. The investigation will further be extended to absorbing layers which are either discontinuous or non-uniform, or both, thus including irregular absorbing clouds.

2. The parameter b, introduced in G I, is dependent, for given values of the absorption coefficients, on k, which itself is dependent not only on the spectral type of a star, establishing its position in the $M - CI$ curve (see fig. 1 in G I), but also on the colour excess (CE), produced by the selective absorption. This CE corresponds to a shift along the abscissa, and k is determined by the straight line connecting the intersections of the projection of this shift on the $M - CI$ curve. Thus k is a function of r_0 because $\mathrm{CE} = a_s r_0$.

The irregular shape of the $M - CI$ curve may be simplified by neglecting the waves appearing for spectral types A and G, and representing it as a smoothed curve; thereby the constant slope for the B stars disappears. This smoothing has been found to have a negligible effect on the relation between r_1 and r_0. The table on p. 382 gives the values of k for the smoothed curve, for different spectral types and CEs.

This table is valid for the photographic $M - CI$ curve. The values for the photovisual curve are smaller by 1, according to formula $k_p = k_{pv} + 1$, given in G I immediately after equation (8).

The dependence of k on the two arguments, spectral type and CE (or r_0), has consequently an influence on the parameter b and, further, on r_1. With k decreasing b increases. The effect on r_1 is obvious.

* *The Distribution of the Stars in Space*, p. 72, 1937.
† *M.N.*, 101, 160, 1941.
‡ *M.N.*, 101, 84, 1941.

Sp / CE	Bo	B2	B5	Ao	A5	Fo	F5	Go	G5	Ko
m										
0·05	15·8	15·5	11·8	8·2	6·8	6·7	6·3	5·8	5·6	4·9
0·1	15·8	15·0	11·4	7·6	6·8	6·6	6·3	5·8	5·4	4·7
0·2	15·0	12·0	9·9	7·2	6·8	6·3	6·2	5·7	5·3	4·6
0·3	13·2	11·3	9·0	7·1	6·6	6·2	6·1	5·6	5·1	4·6
0·4	11·8	10·5	8·3	7·0	6·4	6·1	6·0	5·5	4·8	4·5
0·6	10·2	9·2	7·7	6·7	6·2	6·0	5·7	5·1	4·7	4·4
0·8	9·2	8·5	7·3	6·5	6·0	5·8	5·5	5·0	4·5	...
1·0	8·6	8·0	7·0	6·3	5·8	5·5	5·1	4·8	...	...

(*a*) As to spectral type, let us assume a number of stars of all spectral types in a certain spatial region at a given distance r_0. Then, given an absorption of the type mentioned, the star distances r_1, derived from the CIs, will be spread in depth according to the different parameters b: the blue stars will appear relatively nearer than the red ones. Thus b gives the correction for the reduction to the true distances r_0, and we will call it therefore "absorption modulus" of the corresponding spectral type.

If we keep very nearly to the selective law, represented by the straight line AE in fig. 3 (G I), and accordingly put, for instance, $a = 0{\cdot}25$, $a_s = 0{\cdot}05$ and $r_0 = 1$ kiloparsec (example 1), then the CE $= 0^m{\cdot}05$ and the stars would appear to be spread over an interval of about 226 parsecs from $r_1 = 776$ for Bo stars to $r_1 = 1002$ for Ko stars. If we assume an additional non-selective absorption of $0^m{\cdot}25$ by putting $a = 0{\cdot}5$, $a_s = 0{\cdot}05$ and $r_0 = 1$ kiloparsec (example 2), then the stars are spread over an interval of 250 parsecs from $r_1 = 875$ for Bo stars to 1125 for Ko stars. In the first case b is negative for all spectral types, except K stars; in the second case it changes sign in the late B-type stars ($b = 0$ for $k = a/a_s$, in the present case for $k = 10$). In the first case all stars (except K type) appear to be shifted nearer, but by a different interval; in the second case the B-type stars up to about B8 are shifted nearer, while the other stars appear more distant.

Unfortunately, it seems rather difficult to establish this spreading by observation even in cases where we know the distance of a star group, as with the open clusters, because firstly the range of the distance moduli * of the cluster stars usually overlaps it considerably, and secondly only some clusters show a sufficient range of spectral types. It is, however, intended to investigate this question when many more clusters are available in the survey of galactic clusters, giving the necessary CI and CE.

(*b*) As to the dependence on CE (or r_0), the absorption modulus b is a function of r_0 and increases with increasing r_0. While in the preceding paragraph we have considered the horizontal lines of the table, we have now to inspect the decrease of k in the columns. If we again put $a = 0{\cdot}25$, $a_s = 0{\cdot}05$, then b becomes zero for $k = 5$. The column for, say, G5 stars shows that up to a CE of about $0^m{\cdot}3$ (or r_0 about 6 kiloparsecs) the absorption modulus is negative and the stars will be shifted nearer ($r_1 < r_0$), while for the larger distances $b > 0$ and the stars will appear to be shifted away ($r_1 > r_0$). For the Ko stars b is always positive, while it remains negative for the earlier stars within reasonable distances. If we take the second case, $a = 0{\cdot}5$, $a_s = 0{\cdot}05$, then $b = 0$ for $k = 10$; the B8 stars will appear nearer up to 1 kiloparsec, the B5 stars up to 4 kiloparsecs, the B2 stars up to 10 kiloparsecs, the Bo stars up to 13 kiloparsecs. For the remaining columns b is positive and increases with increasing distance, so that the stars will always appear to be shifted away.

(*c*) If we now consider the whole table we get a picture of the influence of an absorbing layer on the apparent spatial distribution of spectral types, the distribution expressed in distances derived from the observed CI. The true mixture in spectral types will be falsified by the differential shifts of the various spectral types. The amount of this falsification for any absorption coefficients can easily be established with the help of the table.

* *M.N.*, 101, 96, 1941.

3. The above investigations were based on the assumption of a continuous, and uniform, absorption layer. This, however, is the simplest assumption. It may be that the layer does not extend throughout the whole galaxy but is restricted to a smaller volume, say the local star cloud, and we need not even assume that the Sun's position is in the centre of such a layer. The formula

$$\log r_1 = \log r_0 + b r_0 \tag{1}$$

is valid only within the layer, and for all distances outside the layer it will become

$$\log r_1 = \log r_0 + bR, \tag{2}$$

where R is the distance between the Sun and the boundary of the layer in a certain direction. This relation is represented by the curves I and I′ for $b = -0{\cdot}1$ and $b = +0{\cdot}1$ in fig. 1. It is assumed that $R = 1{\cdot}5$ kiloparsec. The absorption modulus b changes its value only within distances up to $r_0 = R$, and the falsification of the spectral-type mixture will then not increase with the distance.

4. If we assume a uniform absorbing cloud at a certain distance and of a certain depth, then the relation between r_1 and r_0 will change according to the position of the star. For stars between Sun and cloud

$$r_1 = r_0. \tag{3}$$

For stars within the cloud

$$\log r_1 = \log r_0 + b(r_0 - R_1), \tag{4}$$

where R_1 is the distance from the Sun to the nearer boundary of the cloud, and for stars behind the cloud

$$\log r_1 = \log r_0 + b d_1, \tag{5}$$

where d_1 is the depth of the cloud in the direction of the star. This relation is represented in fig. 1 by the curves II and II′ for $b = -0{\cdot}1$ and $b = +0{\cdot}1$. It is assumed that there is an absorbing cloud between 1·0 and 1·5 kiloparsec. The spread in the spectral mixture ranges up to about 90 parsecs, a comparatively small value. Assuming a larger selective absorption, as it actually occurs in obscuring clouds, the spread would increase remarkably.

5. When we assume the presence of two or more clouds in the line of vision with clear gaps between them, we have to extend formula (5) by the corresponding members for the consecutive clouds, provided we use the individual values of b for the corresponding clouds. For two clouds the respective equations for stars within and beyond the second cloud are

$$\log r_1 = \log r_0 + b_1 d_1 + b_2(r_0 - R_2), \tag{6}$$

$$\log r_1 = \log r_0 + b_1 d_1 + b_2 d_2. \tag{7}$$

A practical example of this kind is given by the curves III and III′ in fig. 1. The curves are drawn according to an investigation on absorbing clouds near the North America nebula by Müller and Hufnagel.* The photographic and photovisual magnitudes of about 8000 stars in this region were determined and, by a very careful investigation, the presence of two absorbing clouds established. The first of them extends from 75 to 200 parsecs with a total absorption of $0^m{\cdot}6$ ($a = 4^m{\cdot}8$) and no selective absorption, the second one extends from 600 to 800 parsecs with a total absorption of $1^m{\cdot}8$ ($a = 9^m{\cdot}0$) and a selective absorption (or CE) of $0^m{\cdot}27$ ($a_s = 1^m{\cdot}35$) in the densest part of the cloud. Consequently, in the first cloud b is positive and constant for all spectral types (because $a_s = 0$) and all stars are shifted towards the background. The curves III and III′ are identical up to the second cloud. There they are divided because the selective absorption of the second cloud introduces the diversion of the absorption moduli of the different spectral types and the stars begin to spread conformably, up to about 80 parsecs.

* *Zs. f. Ap.*, **9**, 331, 1935.

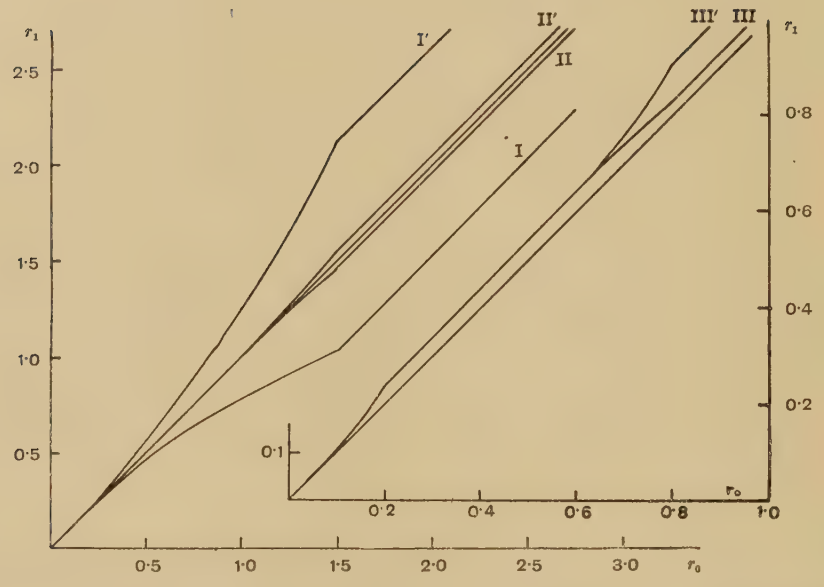

FIG. 1.—*Relation between r_1 and r_0 as affected by various assumptions on absorbing clouds.*

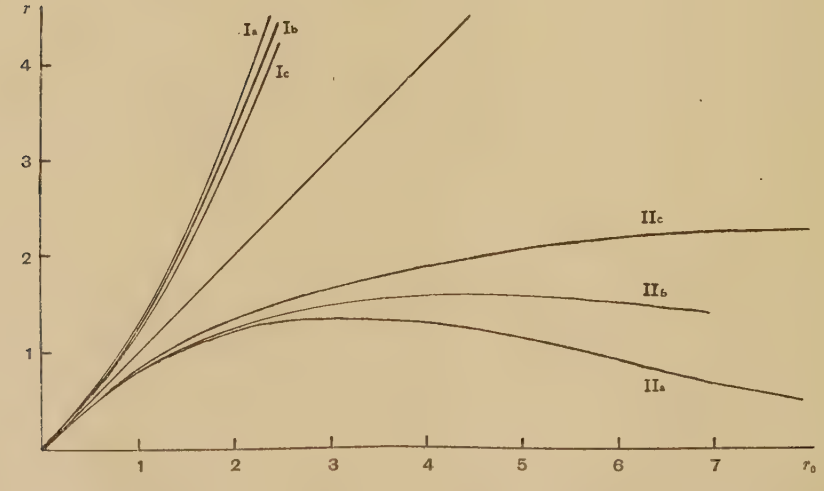

FIG. 2.—*...ion between r_1 and r_0 in a stratum with density varying exponentially.*

Ia : $b = +0{\cdot}1$, $c = +0{\cdot}1$ IIa : $b = -0{\cdot}1$, $c = +0{\cdot}1$
Ib : $b = +0{\cdot}1$, $c = 0{\cdot}0$ IIb : $b = -0{\cdot}1$, $c = 0{\cdot}0$
Ic : $b = +0{\cdot}1$, $c = -0{\cdot}1$ IIc : $b = -0{\cdot}1$, $c = -0{\cdot}1$

The shape of the curves III and III′ is confirmed by, and confirms, the graphs of Müller and Hufnagel's paper. In their figs. 9 to 13 the two authors plot in the upper parts the star number, obtained from observation, against the apparent magnitudes (the curves A, B, C refer to three portions of the obscured region, with total absorptions of $1^m.8$, $1^m.2$ and $0^m.2$, respectively, for the second cloud), and compare the curves with an average curve T of an unobscured normal comparison region. In the lower part of the figures a similar graph is drawn, but with star numbers computed from a standard spectral distribution, under the assumption of the two obscuring clouds. The figs. 9 to 13 refer respectively to different ranges of spectral classes. Only two of them, fig. 12 for F stars and fig. 13 for A stars, can be considered here because, as the authors point out, in the later classes too many giants are mixed up with dwarfs, and a comparison of these graphs with the present investigation (which is valid only for the main sequence, as remarked in G I) would give no reliable results. The B stars are not dealt with in the graphs because "they are not numerous enough." Therefore the curves III and III′ in the present fig. 1 correspond to Ao and F9 stars respectively.

The two authors aimed at obtaining agreement between the influence of the supposed two absorbing clouds on the distribution of star magnitudes as derived from observation and as computed from a standard table. The corresponding two graphs also confirm, however, the influence on the star numbers, due to selective absorption, for different spectral types. According to the curves III and III′ (fig. 1 of this paper) the Ao stars (curve III) are apparently shifted nearer and the F9 stars (curve III′) shifted farther, *i.e.* in an absorbing cloud the A stars must appear more numerous and the F stars less numerous, in relation to a standard distribution. That is confirmed by a comparison of the corresponding upper and lower graphs in figs. 12 and 13. Müller and Hufnagel find (p. 352) that the influence of the second absorbing cloud starts to affect the stars of about 11^m. Fig. 12 (F stars) shows that the observed number of stars fainter than 11^m is smaller than the computed one, and fig. 13 (A stars) shows the opposite, as expected. The influence decreases with decreasing absorption, and the comparison of the curves A and B (corresponding, as mentioned above, to different portions of the cloud with decreasing absorption) with the corresponding curves in the standard graph shows the same effect. (The curves III and III′ are drawn under the assumption of the absorption of curve A; for the smaller absorption coefficients the distance spread would be accordingly smaller, and consequently the difference between the observed and standard star numbers would also be smaller. Curve C was not used because the absorption coefficients are too small to have any appreciable effect.)

6. The fact that an absorbing stratum, if present, is reinforced in many galactic regions by absorbing clouds of various sizes, depths and absorptions makes it impossible to use any one of the developed equations throughout the whole galactic plane. Even equation (7) may be of no use because it does not assume a continuous stratum. If we suppose the general case of such a stratum and some superimposed absorbing clouds in one region, then the equation has the general form

$$\log r_1 = \log r_0 + br_0 + \sum_{n=1}^{n=n} b_n d_n, \qquad (8)$$

with an equation similar to equation (6) for intermediate distances.

7. So far the absorbing layer has been supposed to be of uniform density. We will now assume this not to be the case, because it cannot be considered impossible that an absorbing layer has a maximum density in its centre with density decreasing outwards, for instance according to an analytical law. We will assume this law to be exponential. Then the total absorption coefficient at a distance r_0 has the form

$$a = a_0 e^{cr_0}, \qquad (9)$$

where a_0 is the absorption coefficient in the region of the Sun and c is a constant. The whole total absorption up to r_0 is represented by

$$A = \int_0^{r_0} a_0 e^{cr_0} dr_0. \tag{10}$$

The same form is true for the selective absorption, and the integrated absorptions are

$$A = \frac{a_0}{c}(e^{cr_0} - 1), \qquad A_s = \frac{a_s}{c}(e^{cr_0} - 1). \tag{11}$$

Inserting these values in formulæ (2) and (4) of G I instead of the respective values $a_{pv}r_0$ and $a_s r_0$, we get finally

$$\log r_1 = \log r_0 + \frac{b}{c}(e^{cr_0} - 1). \tag{12}$$

(For c converging to zero formula (12) converges to formula (5) of G I.)

As in G I this family of curves show a maximum for negative values of b, and we see that the sign of c has no influence on this quality. The maximum occurs at

$$r_0 e^{cr_0} = -\frac{\log e}{b}. \tag{13}$$

Fig. 2 shows the deviations of the curves produced by the coefficient c. The values of $+0{\cdot}1$ and $-0{\cdot}1$ correspond very nearly to a real increase or decrease of density of 10 per cent. in 1000 parsecs. A density decrease diminishes the spread in the spectral distribution.

If we first assume a local cloud having its centre in the region of the Sun and filling the local star cloud, then c is always negative and the spectral mixture will be falsified in the same way in all directions. If we suppose, however, a stratum having its centre in the centre of, and stretching throughout, the Milky Way, then this mixture falsification will become very complicated. In the direction to the centre c will be positive and the spectral spread increased; in the direction to the anticentre c will be negative and the spread decreased. (As a matter of fact, an extension of A (formula (10)) beyond the centre would mean a sudden change from $+c$ to $-c$ in the centre and thus represent another complication for the mixture in this direction, but the necessity for considering this case may not arise for a long time because of "the great distances involved." *) In any other direction the resulting equation has no definite solution. If R_c is the distance between observer and cloud centre and r_c the new variable related to the cloud centre, then

$$r_c = \sqrt{R_c^2 + r_0^2 - 2r_0 R_c \cos l}, \tag{14}$$

where l is the angle between the directions of R_c and r_0. The absorption coefficients have then the form

$$a = a_R e^{-cr_c} = a_0 e^{c(R_c - r_c)}, \tag{15}$$

and the final equation is

$$\log r_1 = \log r_0 + b\int_0^{r_0} e^{c(R_c - \sqrt{R_c^2 + r_0^2 - 2R_c r_0 \cos l})} \frac{(r_0 - R_c \cos l)dr_0}{\sqrt{R_c^2 + r_0^2 - 2R_c r_0 \cos l}}, \tag{16}$$

which can be solved only numerically.

It is thus obvious that the presence, or the assumption, of a non-uniformly absorbing layer would make the investigation very difficult.

* Shapley, *M.N.*, **94**, 796, 1934.

Summary

The influence of galactic absorption on many galactic, and even extragalactic, investigations makes it desirable to free our knowledge of its qualities from restrictions imposed by the method of investigation. This is based on knowledge of stellar spectra and total absorption, and is thus limited by the limits of our knowledge of spectra. An extension is possible by the knowledge of CI, penetrating far deeper into space, of the total absorption and of CE.

In the present paper an attempt is made to derive the relation between true star distances and those determined by observed CI, assuming various conditions of interfering absorbing matter. It is shown that the average mixture of spectral types is falsified by this interference, and the nature of this falsification is discussed. An instance, taken from a CI investigation in a region of absorbing clouds, shows good agreement with the deductions of this paper.

In conclusion, I would like to express my thanks to Mr. D. L. Edwards for discussions on this paper.

Norman Lockyer Observatory,
Sidmouth:
1941 *July* 12.

OCCULTATIONS OF STARS BY THE MOON OBSERVED AT THE NIZAMIAH OBSERVATORY, HYDERABAD, DURING THE YEAR 1940

(Communicated by the Director)

(Received 1941 October 6)

TABLE I

No.	Star	Mag.	Phase	Date 1940	U.T. h m s
1	B.D. $-2°$ 5914	7·1	D	Jan. 14	14 40 33·7
2	B.D. $-2°$ 5917	7·7	D	14	14 54 35·7
3	Alger $-2°$ 23^h 12^m 199		D	14	14 54 41·7
4	B.D. $+1°$ 4817	8·5	D	15	14 27 27·7
5	Alger $+2°$ 23^h 52^m 202		D	15	14 40 21·0
6	B.D. $+5°$ 102	8·5	D	16	14 06 49·1
7	B.D. $+14°$ 490	8·8	D	Feb. 15	14 45 26·9
8	B.D. $+14°$ 491	9·5	D	15	14 49 38·9
9	B.D. $+14°$ 492	7·0	D	15	14 57 25·8
10	26 Geminorum	5·1	D	19	18 57 07·6
11	δ Tauri	3·9	D	Mar. 15	14 16 32·9
12	B.D. $+17°$ 716	9·3	D	15	15 32 37·5
13	68 Tauri	4·2	D	15	15 55 20·9
14	B.D. $+18°$ 1141	6·8	D	17	15 40 05·5
15	B.D. $+17°$ 1518	6·7	D	18	14 51 16·9
16	λ Geminorum	3·6	D	18	18 41 40·0
17	B.D. $+15°$ 1775	6·1	D	19	15 22 51·7
18	B.D. $+13°$ 1994	6·8	D	Apr. 16	15 43 49·4
19	B.D. $-15°$ 5626	6·2	R	28	19 47 26·1
20	β Capricorni	3·2	R	28	19 55 12·1
21	B.D. $+13°$ 1940	6·4	D	May 13	15 44 23·1
22	6 Leonis	5·3	D	14	17 37 10·0
23	B.D. $+6°$ 2301	6·5	D	15	15 20 20·0
24	B.D. $-3°$ 3262	7·0	D	17	18 05 48·4
25	B.D. $-3°$ 3263	6·6	D	17	18 05 53·4
26	26 Geminorum	5·1	R	Aug. 28	22 32 22·6
27	13 Piscium	6·5	D	Oct. 13	19 00 45·7
28	14 Piscium	6·0	D	13	20 20 43·6
29	B.D. $-13°$ 5779	7·0	D	Nov. 6	15 50 16·1
30	B.D. $-8°$ 5817	7·0	D	Dec. 5	13 01 39·8
31	B.D. $-5°$ 5894	6·4	D	6	13 08 33·1
32	B.D. $-13°$ 5813	6·6	D	31	13 32 45·6

Two observations (Nos. 6 and 30) were made by T. P. Bhaskaran, two (Nos. 31 and (2) by M. V. V. Sastri and the rest by M. K. Bappu. The instrument used throughout was the 15-inch visual refractor. The corrections to the standard clock were determined from observations with the transit instrument.

The reductions were carried out by M. V. V. Sastri assisted by V. V. Raghaviah. A correction of $+0^h{\cdot}00076$ has been added to the observed time to allow for the error of $1''{\cdot}5$ in the mean longitude of the Moon. No correction, however, has been applied in the case of No. 32, which was observed during the lunation commencing on 1940 December 28.

TABLE II

No.	$\sigma' - \sigma$	$\cos(\rho - \psi)$	$\sin(\rho - \psi)$	No.	$\sigma' - \sigma$	$\cos(\rho - \psi)$	$\sin(\rho - \psi)$
	″				″		
1	−0·2	+0·91	+0·41	17	+0·8	+0·98	−0·22
2	+0·4	+0·89	−0·45	18	−0·4	+0·95	+0·32
3	+1·2	+0·89	−0·45	19	+2·2	−0·99	−0·12
4	−0·4	+0·96	+0·27	20	+1·6	−0·99	−0·14
5	−0·4	+0·95	+0·30	21	+1·0	+1·00	−0·06
6	+3·8	+1·00	+0·03	22	−0·2	+0·41	+0·91
7	[−8·8]	+0·78	+0·63	23	+0·1	+0·96	−0·29
8	−0·2	+0·80	+0·60	24	+0·6	+1·00	+0·07
9	+0·3	+0·96	+0·27	25	−1·0	+1·00	+0·09
10	+0·6	+0·34	−0·94	26	+0·9	−0·87	−0·49
11	−0·8	+0·63	−0·78	27	−1·0	+0·58	+0·82
12	+0·4	+0·09	−1·00	28	+0·2	+0·61	−0·79
13	+0·6	+0·66	+0·75	29	−1·1	+0·96	−0·27
14	−0·2	+1·00	−0·05	30	−2·1	+0·81	+0·58
15	−1·0	+0·88	−0·48	31	−0·8	+0·65	−0·76
16	−3·4	+0·92	−0·39	32	+1·5	+1·00	−0·06

The mean places for 1940·0 of nine stars in the list are given in the following table. For stars Nos. 2, 3, 6, 7, 9 and 12 proper motions have been applied to the co-ordinates given in the catalogue in computing the mean places for use in the reductions. For the remaining stars positions were taken from the *Nautical Almanac*.

TABLE III

No.	R.A. (1940·0)	Dec. (1940·0)	Remarks
	h m s	° ′ ″	
2	23 13 29·66	− 1 54 53·0	A.G. Strassburg 8013 P.M. μ_α +0ˢ·0120 μ_δ 0″.
3	23 13 29·87	− 1 54 50·5	Alger −2°23ʰ 12ᵐ 199 P.M. μ_α +0ˢ·0120 μ_δ 0″.
4	23 57 42·56	+ 2 03 4·4	Alger +1° 23ʰ 56ᵐ 42; +2° 23ʰ 52ᵐ 201.
5	23 57 58·62	+ 2 05 21·9	Alger +1° 23ʰ 56ᵐ 44.
6	0 42 59·89	+ 5 45 42·8	A.G. Leipzig 253 P.M. μ_α −0ˢ·0002; μ_δ +0″·044.
7	2 51 49·16	+14 31 40·8	Bordeaux IV +14° 2ʰ 48ᵐ 47; P.M. μ_α −0ˢ·0053; μ_δ − 0″·056.
8	2 51 56·30	+14 31 42·4	Bordeaux III +15° 2ʰ 52ᵐ 68; Bordeaux IV +14° 2ʰ 48ᵐ 49.
9	2 52 21·53	+14 27 53·0	A.G. Leipzig 870 P.M. μ_α −0ˢ·0028; μ_δ −0″·018.
12	4 20 55·35	+17 20 8·6	A.G. Berlin 1161; P.M. μ_α +0ˢ·0035; μ_δ +0″·002.

Nizamiah Observatory,
Hyderabad, Deccan, India:
1941 *May* 31.

OCCULTATIONS OF STARS BY THE MOON OBSERVED AT MELBOURNE OBSERVATORY IN 1940

J. M. Baldwin, M.A., D.Sc., F.Inst.P.

(Communicated by the Secretaries)

(Received 1941 October 21)

The observations of the occultations in this report were made by the eye and ear method throughout, the clock beating on a relay in the observing room. The telescope used was an 8-inch refractor.

The reductions were computed using the method and reduction elements given in the *Nautical Alamanac* for 1940, a correction of $+0^{h}\cdot00076$ being applied to the observed times. The computations were carried out in duplicate.

TABLE I

No.	Date 1940	Star	Mag.	Phase	U.T. h m s
1	Mar. 21	B.D. +9° 2262	5·9	D	10 23 11·2
2	22	58 Leonis	5·0	D	13 51 38·7
3	Apr. 12	97 Tauri	5·1	D	8 37 29·6
4	May 14	B.D. +12° 2009	6·3	D	10 27 08·0
5	June 13	B.D. +0° 2843	6·2	D	10 53 17·8
6	Sept. 6	B.D. −12° 4134	6·4	D	11 13 40·2
7	9	B.D. −17° 4903	7·5	D	13 16 45·2
8	10	B.D. −17° 5271	6·8	D	12 44 01·0
9	13	*v* Aquarii	4·5	D	9 28 51·8
10	Oct. 6	B.D. −17° 4773	6·0	D	9 44 31·2
11	10	B.D. −12° 5876	7·5	D	10 43 46·4
12	10	B.D. −12° 5890	7·1	D	13 26 40·4
13	11	B.D. −9° 5827	7·1	D	9 15 00·3
14	11	46 Capricorni	5·3	D	9 15 22·0
15	Nov. 8	B.D. −9° 5972	7·5	D	11 43 03·4

TABLE II

No.	p	q	p^2 Unit 0·01	pq	q^2	$\Delta\sigma$	$p\Delta\sigma$	$q\Delta\sigma$
1	+100	− 4	100	− 4	0	+0·2	+0·2	0·0
2	+100	−10	99	−10	1	+0·4	+0·4	0·0
3	+ 78	−63	60	−49	40	+0·4	+0·3	−0·3
4	+100	− 4	100	− 4	0	+1·4	+1·4	−0·1
5	+100	+ 5	100	+ 5	0	+0·3	+0·3	0·0
6	+ 95	+30	91	+29	9	−1·3	−1·2	−0·4
7	+ 84	+54	71	+45	29	−2·2	−1·8	−1·2
8	+ 33	+94	11	+31	89	−1·5	−0·5	−1·4
9	+ 88	+48	77	+44	23	−2·6	−2·3	−1·2
10	+ 99	+14	98	+14	2	−1·4	−1·4	−0·2
11	+ 94	+34	88	+32	12	−1·5	−1·4	−0·5
12	+ 99	−17	97	−17	3	+0·1	+0·1	0·0
13	+ 97	+25	94	+24	6	+6·0	+5·8	+1·5
14	+100	+ 5	100	+ 5	0	−0·4	−0·4	0·0
15	+ 98	+18	97	+18	3	−1·5	−1·5	−0·3

LIST OF ADDITIONS

TO THE

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1941 DECEMBER 1–1942 JANUARY 31

Periodicals received by exchange are not included in this list.

Unless otherwise indicated, the donor is the author or the Institution concerned.

An asterisk (*) indicates that the work is an excerpt.
A dagger (†) indicates that the work has been purchased.

Chichester (Francis):
†Astro-Navigation, Part III. (Observers' Books, No. 5.) Sm. Cr. 8vo. London, 1941

Das (A. K.) and B. G. Narayan:
——: *An Apparent Influence of the Earth on Solar Prominences. 8vo. Calcutta, 1940
——: *Motion of Gases in the Sun's Atmosphere. Parts 1, 2, 3. 8vo. Calcutta, 1940–41

Khan (M. A. R.):
*Synopsis of Meteor Observations in 1940 at Begumpet, Deccan. 8vo. Madras, 1941

University of Minnesota:
The Bruce Proper Motion Survey: The General Catalogue. Part A; Part B, Sections 1, 2. 4to. Minneapolis, 1941

(*W. J. Luyten.*)

Part A. The Proper Motions of 1465 Stars south of Declination −80°. By W. J. Luyten.
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——: The Internal Constitution of the Stars. By Z. Kopal, R. E. Marshak, H. N. Russell, H. Shapley, J. Tuominen. (Astronomical Conference held by the New York Academy). 8vo. New York, 1941

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Magnitudes and Colors of Stars North of +80°. (Carnegie Institution of Washington Publication 532.) 4to. Washington, 1941

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*The Solar Eclipse in the Rigveda and the Date of Atri. 8vo. Calcutta, 1941

Thackeray (A. D.):
*Solar Physics. Cr. 8vo. London, 1940

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*The Effect of Interstellar Matter on Climatic Variation. 8vo. Cambridge, 1939

——: *The Evolution of the Stars. 8vo. Cambridge, 1939
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Jeffreys (Harold), and K. E. Bullen :
*Seismological Tables. 8vo. London, 1940
(*British Association.*)

Jones (V. C.) :
*A Simple Planetarium. 8vo. Toronto, 1940

Macpherson (Hector) :
Biographical Dictionary of Astronomers. 4to. (Typescript), Edinburgh, 1940

Paneth (F. A.) :
†The Origin of Meteorites: being the Halley Lecture delivered on 16th May, 1940. 8vo. Oxford, 1940

Yale University Observatory :
Catalogue of Bright Stars. Second Edition, containing all important data known in January, 1940, relating to all stars brighter than 6·5 visual magnitude, and to some fainter ones; with Appendices containing Galactic Co-ordinates and Indices to the Constellations. By Frank Schlesinger and Louise F. Jenkins. 4to. New Haven, Conn., 1940

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——: 4th edition. Revised and enlarged by L. J. Comrie. 8vo. London, 1941
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Berthoud (F.) :
——: Traité d'Horlogerie. 4to. Paris, 1755
——: Essai sur l'Horlogerie. 2 vols. 4to. Paris, 1763
(*D. Carder-Davies.*)

Greenwich, Royal Observatory :
Catalogue of 20554 Faint Stars in Cape Astrographic Zones $-40°$ to $-52°$.
4to. London, 1941

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*Relativity in the Remotest Heavens. 8vo. London, 1941

Matukuma (T.) :
*On Einstein Effect derived from the Observations of the Total Solar Eclipse of June 19th in 1936. 4to. Tokyo, 1940

[Newton, Sir Isaac] :
Sir Isaac Newton's Parlour brought to America: a Portion of Sir Isaac Newton's Home (1710–1725) re-erected in the United States. Address given by Mrs. Roger W. Babson, October 28, 1939, at the dedication of the Babson Institute Library.
8vo. Babson Park, Mass., 1940
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Sauty (Louis) :
La Mécanique Ondulatoire et les Quanta Electromagnétiques. 8vo. Avignon, 1940

Sohon (F. W.) :
The Stereoscopic Projector. 8vo. New York, 1941
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Voroschiloff (K. A.) :
——: *A Determination of the Type of Tail of Comet 1937*f* (Finsler).
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Clayton (H. Helm):
*A Persistent Solar-rotation Period of 27·26 days. 8vo. Washington, 1941

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O Conselheiro José Luciano de Castro, e o segundo periodo Constitucional Monárquico. 8vo. Coimbra, 1941

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Bulletin No. 13. The Accra Earthquake of June 22, 1939, by N. R. Junner; with Notes by D. R. Bates, and Reports on a Seismological Study of the Earthquake by Ernest Tillotson, and the Engineering Aspects of the Earthquake by C. S. Deakin. 8vo. London, 1941

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Newton (H. W.):
*The Astronomical History of some Old Walling at Greenwich. 8vo. London, 1941

Pearse (R. W. B.) and A. G. Gaydon:
†The Identification of Molecular Spectra. Imp. 8vo. London, 1941

Plummer (H. C.):
†Four-Figure Tables with Mathematical Formulæ. 8vo. London, 1941

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Photometric Atlas of the Solar Spectrum from λ 3612 to λ 8771, with an Appendix from λ 3332 to λ 3637. By M. Minnaert, G. F. W. Mulders and J. Houtgast. Folio. Amsterdam, 1940

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Chichester (Francis):
†Astro-Navigation. Parts 1 and 2. Sm. Cr. 8vo. London, 1941

Ellison (M. A.):
*Problems of the Motions of Solar Prominences. 8vo. London, 1941

Harvard College Observatory:
The Harvard Books on Astronomy, edited by Harlow Shapley and Bart J. Bok. 8vo. Philadelphia, 1941

Earth, Moon and Planets, by Fred J. Whipple.
The Story of Variable Stars, by Leon Campbell and Luigi Jacchia.
Between the Planets, by Fletcher G. Watson.
The Milky Way, by Bart J. Bok and Priscilla F. Bok.

Jones (H. Spencer):
*The 200-inch Telescope. (Twelfth Thomas Young Oration, delivered 1941 May 30). (*The Physical Society.*) Cr. 8vo. London, 1941

Moscow, Physical Institute of the U.S.S.R. Academy of Sciences:
Journal of Physics. Vol. III, Nos. 1–6. 4to. Moscow, 1940

H.M. Nautical Almanac Office:
——: The Nautical Almanac and Astronomical Ephemeris for the Year 1942. Super Roy. 8vo. London, 1941
——: Apparent Places of Fundamental Stars, 1942: containing the 1535 Stars in the Third Fundamental Catalogue (FK_3) of the Berliner Jahrbuch. Imp. 8vo. London, 1941

See (T. J. J.):
Wave-Theory. Book VI. 4to. Lynn, Mass., and London, 1941

Smart (W. M.):
Sea and Air Navigation: being the Halley Lecture delivered on 1941 June 3. Sm. 8vo. Oxford, 1941

Stetson (H. T.):
*A New Laboratory for Cosmic-Terrestrial Research. 8vo. Baltimore, 1941

OCCULTATIONS OBSERVED AT TEDDINGTON DURING 1939 AND 1940

John L. Haughton, D.Sc., F.Inst.P.

(Received 1941 October 20)

Owing to various reasons only four occultations were observed in 1939, and it was not considered worth while publishing these separately. War conditions have prevented any observations being carried out in the latter part of 1940, and so only nine occultations were observed in this year. The results of both years are given in Table I, and the reductions, for which the author is again indebted to Messrs. Scientific Computing Service, Ltd., are given in Table II. The observations were made in the same way as those described in previous reports, and in every case a correction of $+0^{h}\cdot00076$ has been added to the observed time. The reductions are based on the Occultation Reduction Elements given in the *Nautical Almanac*.

TABLE I

No.	Date	Star	Mag.	Ph.	U.T.	P.E.	Remarks
	1939				h m s	s	
149	Jan. 30	B.D. +18° 633	6·0	D	17 39 38·9	0·6	Cloudy.
150	Feb. 6	B.D. −1° 2546	6·2	R	23 46 15·6	?	Doubtful.
151	28	B.D. +18° 1129	6·2	D	22 22 53·9	0·4	
152	Oct. 20	β Cap	3·2	D	17 51 49·2	0·6	
	1940						
154	Jan. 16	B.D. +5° 131	6·3	D	21 25 20·5	0·3	
155	20	δ Tau	3·9	D	21 37 35·1	0·2	
156	20	64 Tau	4·8	D	22 30 45·3	0·3	
157	20	68 Tau	4·2	D	23 28 00·5	2·3	Very doubtful.
158	22	B.D. +17° 1182	5·7	D	19 08 48·5	?	Doubtful.
159	23	λ Gem	3·6	D	20 20 38·8	0·3	
160	Mar. 21	ω Leo	5·5	D	00 28 39·7	?	See note.
162	Apr. 17	B.D. +8° 2289	6·7	D	19 53 49·4	0·4	
163	May 13	A^2 Cnc	5·7	D	20 58 16·9	0·2	

Note.—No. 160 not a sudden disappearance. A rapid loss of brightness over about 0·5 sec. then a slow fading for 3 or 4 secs. The time given is that of final disappearance.

TABLE II

No.	p	q	p^2	pq	q^2	$\Delta\sigma$	$p\Delta\sigma$	$q\Delta\sigma$
149	+100	− 2	100	− 2	0	−0·2	−0·2	−0·0
150	− 95	−31	90	+29	10	+1·7	−1·6	−0·5
151	+ 90	−43	81	−39	19	+0·7	+0·6	−0·3
152	+ 82	+58	67	+47	33	−1·9	−1·6	−1·1
154	+ 55	−84	30	−46	70	+2·1	+1·2	−1·8
155	+ 92	−39	85	−36	15	−0·4	−0·4	+0·2
156	+ 53	−85	28	−45	72	+2·0	+1·1	−1·7
157	+ 39	+92	15	+36	85	−0·5	−0·2	−0·5
158	+ 33	−94	11	−31	89	+1·2	+0·4	−1·1
159	+ 92	−40	84	−37	16	−2·4	−2·2	+1·0
160	+ 52	−85	27	−44	73	−0·3	−0·2	+0·3
162	+ 61	+79	37	+48	63	−0·5	−0·3	−0·4
163	+ 99	+12	99	+12	0·1	+0·6	+0·6	+0·1

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